PROMOTING CONCEPTUAL UNDERSTANDING OF ALGEBRA USING VIRTUAL MANIPULATIVES: A STUDY OF FORM 4 STUDENTS

Wee Kim Teck
Jabatan Matematik
ktwee@rocketmail.com

Abstract

This study was conducted to explore students’ perceptions of the usefulness of virtual manipulatives in promoting conceptual learning of algebra based on the constructivist learning theory (Bruner, 1996), which focused on the use of mathematical representations in conceptual understanding. In this study, virtual manipulatives was used as the instructional method in learning algebra among Form 4 secondary school students. Fifteen Form 4 students were interviewed regarding their conceptual understanding of algebra and whether virtual manipulatives promoted their conceptual understanding of algebra. Content analysis of interview data showed that the use of virtual manipulatives was useful in promoting students’ conceptual understanding of algebra. In conclusion, implications of the findings and suggestions for further research are put forward.

Keywords: Virtual manipulatives, mathematics, conceptual understanding, Form 4 students
Abstrak


*Kata kunci:* manipulatif maya, matematik, pemahaman konseptual, pelajar Tingkatan 4

Introduction

Algebra is a branch of mathematics which focuses on the manipulation of mathematical and algebraic symbols. However, these symbolic representations are abstract in nature. Hence, for most students, the learning of algebra is difficult and boring (Ignacio, 2006). Presently, the main obstacle concerning the learning of algebra in Malaysian schools is the effectiveness of mathematics instructions. The factors that affect the effectiveness of mathematics instructions are: firstly, the teacher-centred classroom instruction (Effandi Zakaria & Zanaton Iksan, 2007; Koh & Hong, 2007); secondly, the sluggishness of teachers in adopting information communication technology (ICT) in mathematics instructions (Mokhtar Nawawi, Ahmad Fauzi Ayub, Wan Zah Wan Ali, Aida Suraya Yunus, & Rohani Ahmad Tarmizi, 2005); and lastly, the influence of the examination-oriented teach-to-test culture (Lim & Hwa, 2006; Yahya Hasan, 2005). These factors have promoted the rote and procedure learning of algebra which emphasized on symbolic manipulation over concrete manipulation without developing much
conceptual understanding on algebra (Effandi Zakaria & Zanaton Iksan, 2007).

Manipulatives have been recommended by the educators in the West to be used in classroom teaching (National Council of Teachers of Mathematics, 1989). For instance, physical manipulatives have long been used to facilitate students’ conceptual understanding and problem solving skills (Durmus & Karakirik, 2006). However, recent advancements in ICT have opened up opportunities in promoting the use of virtual manipulatives to improve mathematics instructions (Moyer, Bolyard, & Spikell, 2002), to assist teachers in enhancing their teaching skills, and to improve the learning experience of students (Yahya Hasan, 2005). Virtual manipulatives is fast emerging as an important supplement to the present classroom instruction in improving students’ learning of algebra.

Since a review of the literature shows that there is a still lack of research on virtual manipulatives in Malaysian schools, this study aimed to investigate students’ perceptions of the usefulness of virtual manipulatives as a tool in promoting conceptual understanding in learning algebra. Currently, rote learning is widely practiced in the learning of algebra in Malaysian schools (Effandi Zakaria & Zanaton Iksan, 2007 reference). In addition, this study hoped to promote interest in the use of virtual manipulatives and to encourage school teachers to use virtual manipulatives as an alternative form of instruction in classroom teaching.

**Purpose of the Study**

This study was designed to gain further knowledge on the use of virtual manipulatives in the learning of algebra by conceptual understanding. In order to achievement this aim, this study intended to determine (a) the usefulness of virtual manipulatives in promoting students’ conceptual understanding of algebra, and (b) how virtual manipulatives promoted students’ conceptual understanding of algebra.

**Literature Review**

Conceptual understanding generally refers to students' ability to apply their knowledge on solving mathematical problems using various modes of representations of mathematical concept and to explain the solution without relying solely on restating the mathematical rules and procedure (Bolyard, 2005). Mathematics learning has been experiencing
a gradual change from rote and procedures learning by symbolic manipulation to conceptual understanding by building a system of mental representations of concepts (The National Council of Teachers of Mathematics, 2000; Pusat Perkembangan Kurikulum, 2004).

The constructivist learning theory based on the work of Bruner (1966) has become increasingly popular as a guiding theory for effective learning. According to Bruner (1966), an individual can think about a particular idea or concept at three different stages: (a) enactive; (b) iconic; and (c) symbolic. The theory focuses on the way learners interact with and create meaning from their environment and experiences progressively when they move from the concrete stage to the abstract stage.

However, having the ability to move from one stage to the next as in the progressive stages was insufficient to ensure understanding of a concept. Many researchers have noted that mathematical representations can play the role in the development of conceptual understanding. Hence, for conceptual understanding to evolve, students need to be able to perform two additional tasks: (a) translation between multiple representations of a concept; and (b) interaction between internal representation and external representation of a concept.

Lesh, Landau, and Hamilton (1983) identified five distinct types of representation systems (Figure 1) that occur in mathematics learning and problem solving. They emphasized the importance of connection and translation among algebra representations to solve problems.
Reys, Lindquist, Lambdin, and Smith (2007) suggested that students should experience many different representations of mathematical concepts. Only after the students have been exposed to different external representations like words and symbols representation of a concept, would they be able to internalize the concept when they achieve the stage of interaction between internal and external representation (Figure 2).
The above reviews have demonstrated how mathematical concepts can develop through: first, moving progressively from the concrete stage to the abstract stage in mathematics learning; secondly, encouraging the student to be actively involved in the use of multiple representations in mathematics; and thirdly, emphasizing on connections and translations among different mathematical representations.

Virtual manipulatives refer to the dynamic visual representations of concrete manipulatives (Spicer, 2000). They are computer software programmes which emulate physical manipulatives by keyboard operation instead of physical action on the three-dimensional objects. Previous studies showed that students who worked with virtual manipulatives demonstrated higher achievement and better attitudes toward mathematics (Namukasa, Stanley, & Tuchtie, 2009), significant improvement of students’ conceptual knowledge in fractions (Reimer & Moyer, 2005), and students using virtual manipulatives outperformed students using physical manipulatives in learning fractions (Suh, 2005). Hence, the present study aimed to explore students’ perceptions of the usefulness of virtual manipulatives in conceptual learning of algebra.
Method

Participants
This study was carried out in three secondary schools in Kuching. A total of 207 Form Four students participated in this study. Due to the time constraint, only fifteen of them were selected for interview. The selection for interview was not random but conveniently selected to have a balanced representation from all the schools.

Instrumentation
This research used virtual manipulatives as the method of instruction. The virtual manipulatives (Figure 3) used in the study were developed based on the guidelines and recommendations from past studies such as Moyer et al. (2002), and Durmus and Karakirik (2006).

A conceptual understanding task sheet on quadratic expansion was used to facilitate the collection of qualitative data. The conceptual task sheet consisted of three items respectively. Each item required the students to present the solution to the problem in three different representational forms: (a) pictorial form – drawing algebraic tiles, (b) table form – using algebraic symbol table, and (c) written symbols form – using algebraic symbols. The items were developed based on the
systems of representations and the development of mathematical concepts literature (Goldin & Shteingold, 2001) as shown in Figure 4.

<table>
<thead>
<tr>
<th>Solve the problem using 3 different representational forms:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Multiply (2x + 3) and (x + 1)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>a) AlgebraTiles</th>
<th>b) Symbol Table</th>
<th>c) Algebraic Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="AlgebraTiles" /></td>
<td><img src="image" alt="Symbol Table" /></td>
<td>( ) ( )</td>
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<tr>
<td>Ans:</td>
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*Figure 4. Example of the Item in Conceptual Understanding Task Sheet*

Due to time constraint, pilot study was not done on the reliability of the conceptual understanding task sheet. However, six individuals who were identified as expert reviewers based on their background and knowledge of mathematics education conducted an analysis on the items. The results from the expert review indicated that the test items were at moderate complexity, which fulfilled the required level of difficulty in measuring students’ algebraic concept and can be used in this study.

**Procedure**

The learning of mathematics by conceptual understanding emphasized on progressive learning from the concrete stage to the abstract stage. Hence, during the one week learning session, the students were taught how to a) make use of the virtual algebra tiles to represent / to convert the abstract symbols into concrete tiles, b) manipulate with the tiles to solve some simple problems and c) convert back the concrete tiles to the abstract symbols. At the end of the session, they were asked to complete the conceptual task sheet as mentioned above and followed by standardized open-ended interviews.
The questions prepared for the interviews were of types like experience, knowledge and feeling base on their works on conceptual task sheet with follow up questions and probes like detailed oriented questions, elaboration questions and clarification questions (Patton, 2002). The interviews took place face-to-face and each interview was audio-taped and transcribed verbatim.

**Data Analysis**

The collected data were analysed using the constant comparative method (Maykut & Morehouse, 1994). The analysis began by identifying the units of meaning from the interview transcripts. The units of meaning were then grouped into categories where the rules of inclusion for each category as well as the examples were prepared and listed down as in Table 1.

<table>
<thead>
<tr>
<th>Category</th>
<th>Rule of inclusion</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Novelty effect.</td>
<td>Students found the algebra tiles to be a new experience, which could motivated the students toward learning algebra.</td>
<td>“The algebra tiles is easy to use and I would like to share my experience with my friend in the other class.”</td>
</tr>
</tbody>
</table>

After preparing the lists of categories, rules of inclusion, and examples, the researcher coded all the interview transcripts.

**Findings**

The interview responses of the students were content analysed and the findings are as follow:

(a) **The usefulness of virtual manipulatives in promoting students’ conceptual understanding of algebra**

In analyzing the interview transcripts, first, the researcher was interested to know whether virtual manipulatives was useful on students’ conceptual understanding of algebra. The analysis of the students’ work
revealed that the use of virtual manipulatives has created a new learning experience that might motivate the students by drawing their interest to learn algebra. This consistent theme was the novelty effect of using the virtual manipulatives as the instructional tool. A student commented: “I have a good friend in the other class and I feel that it is good to let him know the other way of doing quadratic expansion. I am sure he will enjoy it [virtual manipulatives]” (Student 8).

The analysis also revealed that students that prefer solving problems using algebra tiles benefited from virtual manipulatives. Another consistent theme was preferences over virtual manipulatives. A student’s comment that showed the preferences was: “I like using algebra tiles. It is simple and easy to understanding, especially when dealing with negative tiles. Algebraic symbol is difficult when come to negative sign.” (Student 11)

**(b) How virtual manipulatives promoted students’ conceptual understanding of algebra**

Next, the researcher was interested in how students explained their mathematical thinking and justified their work. This would contribute to the main research interest which focused on conceptual understanding of algebra. Hence, the researcher was interested to know the ability of the students in using mathematical representations, that is, whether the students used various representational forms to explain their reasoning or they only relied on stating rules and procedures when solving the problem.

The analysis of the students’ work revealed that there were evidences to show that students using virtual manipulatives were able to work by using various representational forms (pictures, tables, and symbols). One theme was that the virtual manipulatives was helping the students to explain the solution to the problem using various representations of concept. An example of this theme was commented as: “After I have done with the drawing, I change all the tiles [algebraic tiles] to symbols [algebraic terms]. Then I put all the symbols [algebraic terms] in each of the four boxes in the symbol table. Then I add up all the four symbols [algebraic terms] inside the symbol table. So I get the answer” (Student 9).

The analysis further revealed that students showed confidence in solving the problem. Another consistent theme was that students showed an improvement in algebraic skill by using one or more
representational forms to counter check their work. A student’s comment that reflected the improvement was: “I have to wait 3 times, first algebraic symbol, then check with symbol table, and finally I check again with algebraic tiles, and I got the same answers” (Student 4).

Discussion

In this study, initial findings revealed that virtual manipulatives was useful as an alternative instructional tool that provide new learning experience to the students and has drawn their attention in the interest of learning algebra. The findings also revealed that students were able to solve the quadratic expansion problems as well as to facilitate, explain and counter check/self evaluate their work using various representational forms. This was in accord with the report by Moyer, Niezgoda, and Stanley (2005) that students were able to represent their thinking in various representational forms using virtual manipulatives. This ability to move flexibly between different representations of a concept indicated conceptual understanding of that concept (Lesh, Post, & Behr, 1987).

Conclusion

Summary

This study investigated the use of virtual manipulatives that was fast emerging in the literature as an alternative classroom instruction to facilitate the conceptual development of the students. The findings from this study provided some positive results on the use of virtual manipulatives in learning algebra. The usefulness of virtual manipulatives was demonstrated by the novelty effect in using a new instructional tool that most of the students have never seen before. Likewise, the conceptual learning of algebra using virtual manipulatives has shown improvement in algebra and was indicated clearly by students’ confidence in solving the problems using multiple representations of the concept. However, virtual manipulatives should not make the traditional classroom instruction obsolete but instead be used as a supplementary instructional tool to enhance the classroom teaching and learning. Although the use of virtual manipulatives has been enhanced with progress in ICT, more research is needed to fully understand the various issues surrounding virtual manipulatives instruction. The findings of this study contribute to the body of research exploring this question.
Implications
Two major contributions can be drawn from this study. In terms of theoretical contribution, the results of this study supported the concrete to abstract learning theory by Bruner (1966) and the work of Lesh et al. (1983) which showed that manipulatives could be used to promote the conceptual understanding of algebra. Likewise, the study also confirmed that the ability to translate between different mathematical representations of a concept was the key to learning by conceptual understanding. This result was consistent with the work by Reys et al. (2007). In terms of practical contribution, although Domino’s (2010) study reported that manipulatives has positive effect in primary school on mathematics achievement, the finding in this study found that virtual manipulatives was also effective in secondary school as well. Hence, teachers should encourage and motivate students to explore new algebra concepts by focusing more on conceptual learning using virtual manipulatives rather than on rote and procedure learning of algebra in class. The findings of this study could be used as references by the students and the teachers in promoting the interest in the learning of algebra.

Suggestions for Further Research
Future research can be carried out base on the results of this study by focusing on a) longer time for treatment and follow up studies on students’ retention skills, b) replicating the study using larger sample involving more schools and students from other forms beside form four classes, and c) exploring different mathematics contents beside algebra like geometry and trigonometry.

References


