ABSTRACT

Educators have been talking much about constructivism as the learning theory for mathematics education since the last quarter of the last century. Has this new philosophy improved students’ performance in mathematics?

This paper proposes scaffolding as a teaching strategy to enhance mathematics learning in the classrooms. Scaffolding is formulated from Vygotsky’s concept of the zone of proximal development. It emphasizes active participation or a greater degree of control from students over their learning. For successful scaffolding, five key features need to be addressed. These are:

1. Students explain and justify their solutions.
2. Teachers continuously assess students’ understanding.
3. Teachers take into consideration students’ perspectives.
4. Scaffolding tailor to the needs of students.
5. Students take up or use the scaffolding.

When the scaffolding tendered is tailored to the needs of a student tackling a meaningful and challenging task, the student will be able to accomplish the task, which is otherwise impossible. However, teachers need to change their role in the classroom from the sole source of mathematical knowledge to facilitators in the development of students’ mathematical constructions, while employing scaffolding.

Introduction
The 1990s had been a period of great change for mathematics education. A theory of learning called constructivism emerged. New curriculum documents were shaped by educators, which place more emphasis on mathematical constructions, rather than on contents. However, these educators avoided making recommendations about teaching approaches or strategies, which could help realizing this emphasis. Scaffolding as a teaching strategy could help to materialize the dream of these educators. In mathematics lessons where scaffolding is employed as a teaching strategy, the conventional assumptions about what it means to know mathematics are challenged. It becomes clear to the teachers that teaching is not only about teaching what is conventionally called content, but also facilitating students’ mathematical constructions. Thus it is necessary for both teachers and students assuming different kind of roles and responsibilities to do different sorts of activities together.

**Constructivism**

Constructivism emerged in the early 1980s, and owes its debt to Piaget’s Developmental Psychology and the Vygotskian School of Learning. Von Glasersfeld (1995) lays out two basic principles which convey the flavour of his radical constructivism:

- *knowledge is not passively received but built up by the cognizing subject;*
- *the function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality.* (p. 18)

These principles imply that learning has to do with more than what is experienced through the senses. Lerman (1983) describes an example where students were asked to find a fraction between 1/2 and 3/4. One of the students gave 2/3 as an answer. The student insisted that this solution was correct and easy to find and refused to accept another way of getting a solution since 2 is between 1 and 3 and 3 between 2 and 4, which is true. The teacher then asked the student another question – find a fraction between 1/2 and 1/3. This time, the student was unable to reason out a solution using the above method, since there is no number between 1 and 1 or 2 and 3.
‘If the teacher at once reacts by saying that [students’] ideas are wrong and tells them what is considered right, the students may indeed adopt the suggestion, but the reason why it is considered better may not be understood. It would seem more efficient to present the students with situations where the lay theory they have been using does not work.’ (Von Glasersfeld, 1995, p. 87)

Radical constructivism is increasingly being criticized for its limitations as a learning theory. Those educators who adhere to the Vygotskian School of Learning (Confrey, 1990, Steffe & Kieren, 1994, Lerman, 1996) suggest the extension of radical constructivism to social constructivism by incorporating ‘intersubjectivity’, which views mathematics learning as both a collective human activity and an individual constructive activity, rather than just an individual element for radical constructivism. Confrey (1990) says that

‘... the constructive process is subject to social influences. We do not think in isolation; our choice of problem, the language in which we cast the problem, our method of examining a problem, our choice of resource to solve the problem, and our acceptance of a level of rigor for a solution are all both social and individual processes.’ (p. 110)

In other words, there are two faces of mathematics. These are mathematics in students’ heads and mathematics in the students’ environment. The main concern of social constructivists is how to account for mathematics learning in the students’ environment.

**Implications for Instruction**
Since constructivism is a way of thinking about knowledge and the act of learning, what are its implications for instruction in the classroom?

‘An instructor should promote and encourage the development for each individual within his/her class of a repertoire for powerful mathematics constructions for posing, constructing, exploring, solving and justifying mathematical problems and concepts and should seek to develop in
students the capacity to reflect on and evaluate the quality of their constructions.’ (Confrey, 1990, p. 112)

The following list, which should by no mean be treated as complete, contains a few, but important implications.

1. Students should not be treated as ‘empty vessels’ or ‘blank slates’. We should advocate an interactive classroom context by encouraging group work or whole class student-teacher discussion. By doing so, students learn to explain and justify the legitimacy of their solutions. By doing so, they are given a greater share of their work, and hence a greater responsibility for their learning (Bickmore-Brand & Gawned, 1990, Cobb, Wood & Yackel, 1991, Cobb, Wood, Yackel & McNeal, 1992).

2. Encourage reflective language such as
   ‘How do you think ...?’
   ‘How might ...?’
   ‘What are we doing?’
   ‘Why is it that ...?’
   ‘Think ...’
   ‘Do you remember ...?’
   ‘Do you know ...?’
   ‘I don’t believe ...’
   ‘What ... if ...?’ (Bickmore-Brand & Gawned, 1990, Confrey, 1990)

3. Teachers should listen to students and observe the learning process. (Steffe & Kieren, 1994, p. 724)

4. Learning activities should be related to student’s prior knowledge and interests. When student has difficulty, teachers should provide help tailored to the needs of students (Bickmore-Brand & Gawned, 1990, Confrey, 1990, Steffe & Kieren, 1994).

5. As knowledge is growing, we should formulate activities showing the interrelatedness of different concepts. One of the problems contributes to poor performance in mathematics by students is a lack of continuity in their learning. So, with the right activities, teachers will be able to make
their students conscious of their learning so that they are able to reflect on their own and others’ work (Good, Mulryan & McCaslin, 1992).

**Scaffolding**

Vygotsky’s school of thought probably has the most profound influence on the formation of the concept of scaffolding in the cognitive development of a child (Greenfield, 1984, Rogoff & Gardner, 1984, Stone, 1993). Vygotsky conceptualizes the idea of the zone of proximal development. He says that children who by themselves are able to perform a task at a particular cognitive level, in cooperation with others and with adults will be able to perform at a higher level, and this difference between the two levels is the child’s ‘Zone of Proximal Development’. Vygotsky claims that

‘Every function in the child’s cultural development appears twice, on two levels. First on the social, and later on the psychological level; first, between people as an interpsychological category and then inside the child as an intrapsychological category.’ (1978, p. 128)

The process by which inter becomes intra is called internalization and involves more than the endowment of the child and more than the child can accomplish on his or her own, but it occurs within the child’s zone of proximal development. Hence Vygotsky proposes that the cognitive development in a child is social, which involves another person and the society as a whole. In other words, social interaction taking the form of dialogue or cues or gestures, plays an important role in concept formation.

Another important factor that determines the cognitive development of students is the characteristics of the task that is assigned to them.

‘The tasks with which society confronts an adolescent as he enters the cultural, professional, and civics world of adults undoubtedly become an important factor in the emergence of conceptual thinking. If the milieu presents no such task to the adolescent, makes no new demands on him,
and does not stimulate his intellect by providing a sequence of new goals, his thinking fails to reach the highest stages, or reaches them with great delay.’ (Vygotsky, 1986, p. 108)

The task should be related to students’ everyday experiences so that they have something which they are familiar with to reflect on. At the same time the task should not be too simple or it will not ‘stretch’ the thinking of students.

Wood, Bruner and Ross (1976) introduced the word scaffolding for the first time in their article ‘The Role of Tutoring in Problem Solving’. They believe that the acquisition of skills by a child is an activity in which the readily relevant skills are combined and ‘bent’ into ‘higher skills’ to meet new, more complex task requirements. This activity can only be successful through the intervention of a tutor, which will result in much more than just modelling and imitation.

‘More often than not, it involves a kind of “scaffolding” process that enables a child or novice to solve a problem, carry out a task or achieve a goal which would be beyond his unassisted effort. This scaffolding consists essentially of the adult “controlling” those elements of the task that are initially beyond the learner’s capacity, thus permitting him to concentrate upon and complete only those elements that are within his range of competence. The task thus proceeds to a successful conclusion. We assume, however, that the process can potentially achieve much more for the learner than an assisted completion of the task. It may result, eventually, in development of task competency by the learner at a pace that would far outstrip his unassisted efforts.’ (Wood, Bruner & Ross, 1976, p. 90)

They also outline six key functions of scaffolding:

1. Recruitment: engaging the student in a meaningful and interesting task;
2. Reduction in the degree of freedom: breaking the task into manageable components;
3. Direction maintenance: keeping the students on-task and on-track to a solution;
4. Marking critical features: accentuating key parts of the task;
5. Frustration control: decreasing the stress of the task but not so far as to create total dependency on the tutor; and
6. Demonstration: The tutor imitates attempted solution by the tutee, hoping that it will be imitated back by the tutee in a more appropriate form.

Greenfield (1984) defines the scaffold for building construction as follow:

‘The scaffold, as it is known in building construction, has five characteristics: It provides a support; it functions as a tool; it extends the range of a worker; it allows the worker to accomplish the task not otherwise possible; and it is used selectively to aid the worker where needed.’ (p. 118)

Based on this definition, she puts forward the following idea of the scaffolding process in a learning situation.

‘… the teacher’s selective intervention provides a supportive tool for the learner, which extends his or her skills, thereby allowing the learner successfully to accomplish a task not otherwise possible. Put another way, the teacher structures an interaction by building on what he or she knows the learner can do. Scaffolding thus closes the gap between task requirement and the skill level of the learner …’ (p. 118)

Cambourne (1988) highlights the common interactions in scaffolding as:

Focusing on a gap to bridge in child skills/knowledge to accomplish a task.
Extending by raising the skill level: asking questions like ‘What else will you (would you, could you) do?’ when the teacher is satisfied with the performance of the child.
Refocusing by encouraging clarification and justification by asking questions like ‘Is this what you are trying to say (do, write) or is it something else?’
when the teacher is confused or unclear about what the child is doing or saying.
Redirecting by offering new resources if there is a mismatch between the child’s intent and the message or in the expectations which the teacher holds for the child.

No matter how one defines scaffolding, it is ‘a metaphor for the temporary framework experts help create for novices in their attempts to solve problems’ (Lehr, 1985). Scaffolding exhibits the following key features:

- Scaffolding has the capacity to enhance the potential of an individual within his zone of development;
- It requires a meaningful and challenging task;
- It emphasizes active participation of a learner in tackling a task; and
- Scaffolding is developmental.

Research Project

A research was conducted in a high school in New Zealand. Two mathematics teachers, 31 students of third form (around 13 years of age) and 48 students of sixth form (around 16 years of age) participated in this research. The mathematics programs for the third form and the sixth form of the school from May 1997 until the end of 1997 were adopted. One of the purposes of the research was to seek for key features of successful scaffolding.

The research was collaborative, and was guided by a naturalistic inquiry and an action research philosophy. Three formal methods of data collection were used: (1) video recording of lessons, (2) running records of observations and discussions in the classroom, and (3) audio taping of the discussion sessions and interviews with students.

A framework was developed to analyze the data. The framework involved an initial identification of patterns of interest. Segments of tapes where these patterns were
observed were transcribed throughout the research and grouped under these patterns together with the running records. This was followed by episode-by-episode analyses, which were guided by a number of themes. Finally, the comparative analysis involved a meta-analysis of the episodes to develop an overview of the progress under the various themes. Those chronological analyses served as the bases for developing the case studies of the two participating teachers. These case studies were necessary and served as evidence for this research.

**Discussion**

Prior to the teachers participating in this research, they taught mathematics by the traditional method, which one of them described as

‘...the way [he] was taught mathematics in a school similar to this, which was - would be notes, examples and working problems.’

A typical lesson consisted of either reviewing or introducing a new concept through examples, during which he presented students with step-by-step instructions. This was followed by other activities such as giving notes, assigning working problems from the textbook and monitoring students while they were working on those problems.

Besides the discussion sessions, during which the participating teachers and the researcher discussed issues such as ‘rich problems’ and ‘scaffolding techniques’ before this research commenced, the teachers also attended a one-day problem solving workshop organized by the Otago Mathematics Teachers’ Association during the first week of July 1997. Other than this, the assistance that they got came from informal discussions with the researcher, both in the classroom and outside the classroom. There was no fixed agenda for such discussions. Any issue of concern arising from the lessons could be the topic for discussion. During these discussions, the researcher informed the teachers of current theories, ideas and research findings required to resolve the issues. The negotiated outcomes were then implemented in the subsequent lessons.
Successful Scaffolding

The following questions were identified to successfully scaffold students through different stages in solving a mathematical problem.

<table>
<thead>
<tr>
<th>SCAFFOLDING QUESTIONS</th>
<th>PURPOSES</th>
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<tbody>
<tr>
<td><strong>GETTING STARTED</strong></td>
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<tr>
<td>What are the important ideas here?</td>
<td>Encourage careful reading;</td>
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<tr>
<td>Can you rephrase the problem in your own words?</td>
<td>Understand vocabulary.</td>
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<tr>
<td>What is this asking us to find out?</td>
<td>Bring the language to the appropriate level for the student.</td>
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<tr>
<td>What information is given?</td>
<td>Identify and clarify the problem.</td>
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<tr>
<td>What conditions apply?</td>
<td>Sensible prediction may help.</td>
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<tr>
<td>Anyone want to guess the answer?</td>
<td>Eliminate inappropriate starting strategies.</td>
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<tr>
<td>Anyone see a problem like these before?</td>
<td>Use a similar approach or strategy to one previously used successfully.</td>
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<td>What strategy could we use to get started?</td>
<td>Use a strategy gives an entry point.</td>
</tr>
<tr>
<td>Which one of these ideas should we pursue?</td>
<td>Get ideas for possible ways to solve a problem.</td>
</tr>
<tr>
<td><strong>WHILE STUDENTS ARE WORKING ON THE PROBLEM</strong></td>
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<tr>
<td>Tell me what you are doing here?</td>
<td>Help students clarify and confirm their own thinking.</td>
</tr>
<tr>
<td>Why do you think of that?</td>
<td>Help students avoid wild goose chase and keep the end point in view.</td>
</tr>
<tr>
<td>Why are you doing this?</td>
<td>Identify appropriate level of support.</td>
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<tr>
<td>What are you going to do with the results once you have it?</td>
<td>Help students overcome barriers by allowing them to propose alternative strategies.</td>
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<tr>
<td>Why do you think that stage is reasonable?</td>
<td>Redirect back to an earlier stage.</td>
</tr>
<tr>
<td>Why is that idea better than that one?</td>
<td>Help students ensure their understanding.</td>
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<tr>
<td>You’ve been trying that idea for 5 minutes. Are you getting anywhere with it?</td>
<td></td>
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<tr>
<td>Do you really understand what the problem is about?</td>
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<tr>
<td>Can you justify that step?</td>
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</tbody>
</table>
Are you convinced that bit is correct? work makes sense.
Can you find a counter example?

AFTER STUDENTS THINK THEY ARE FINISHED

Have you answered the problem? Help students see the full solution.
Have you considered all the cases? Check work is logical.
Have you checked your solution? Importance of process as well
Does it look reasonable? as answer.
Is there another solution? Look for a better solution/
Could you explain your answer to the class? highlight features of a variety
Is there another way to solve the problem? of strategies.
Can you generalise the problem? Challenge early finishers to
Can you extend the problem to cover different and situations? obtain a more powerful solution.
Can you make up another similar problem? Motivate students to find and solve their own problems.

Five key features were identified for successful scaffolding. These are:

- Students explain and justify their solutions.
- Teachers continuously assess students’ understanding.
- Teachers take into consideration students’ perspectives.
- Scaffolding tailor to the needs of students.
- Students take up or use the scaffolding.

**Explain and justify solution:** At the beginning of this research, students either did not response to the teachers’ scaffolding questions, or students’ responses were confined to brief phrases and in single disconnected sentences. The teachers then amplified what they supposed their students might have meant. As the teachers emphasized students explaining and justifying their solutions, the students improved drastically in doing so. As such, the teachers and their students often had a shared understanding on an issue, which ensured successful scaffolding.
Assess students’ understanding: The teachers constantly assessed students’ understanding before tendering scaffolding tailored to the misunderstanding or the lack of understanding of the students.

Students’ perspectives: Students’ solutions were taken into consideration when devising instruction or tendering scaffolding. As such, scaffolding was successful since the starting point was something familiar to the students.

Students’ needs: At the beginning of this research, the teachers would answer their own questions if the students did not response to these questions. In other words, the scaffolding often resulted in the imposition of their methods on students. As the students improved in explaining and justifying their solutions, they and their teachers often had a shared understanding on an issue. Hence the teachers were able to assess the needs of their students and tendered the right scaffolding to help them.

Employing the scaffolding: Students needed to use the scaffolding on the spot so that the messages from their teachers could be conveyed successfully to them during the scaffolding process. If the scaffolding was left to students without employing it, it was likely that the scaffolding would be ignored, especially if students had a method in hand to tackle a problem.

Reconceptualizing the teacher’s role

As mentioned earlier, the participating teachers previously taught mathematics as a procedure-oriented subject, during which they presented students with step-by-step instructions. Then they assigned activities for students to complete individually. The students’ solutions were evaluated and indicated whether they were right or wrong by the teachers. In the end, the students were given notes, ‘whether they liked it or not’. In other words, the teachers were the sole source of knowledge in the classroom.

As the teachers learned from the conflicting situations in the classroom and changed their practice in teaching mathematics in this research, they started questioning their role as the sole source of knowledge in the classroom. They were more inclined to lead a lesson from students’ responses.
'Such lessons] made me more aware of [the students’] side of the classroom if you like. They’re sitting there. I’m up front raving. They’re not learning much … You know, what are the students doing? … If I was one of them, what am I going to do during this period? … What sort of active participation am I going to be taking in the lesson? And so it’s made me think a little more about that. I guess that’s sort of involved with their development of ideas. You’re more interested with what way they may take of this. Not which way I want them to go or not which way might I take it, but what they may pick up. And you can’t know until you actually get into the lesson and suddenly someone comes up with a sort of tangent idea and you think I haven’t thought of that, but that’s actually quite good and you’ll go with that. Whereas before … I’d say, well that’s a good idea but that’s not where I want you to be going, so we won’t go down that path.’

When the teachers encountered students’ misunderstanding, they seldom tried to impose their methods on them. One of the teachers said

‘[You] either keep them going on that path or put up a wall in that path. And that’s difficult because sometimes you can’t think of one and you need to think of an example. I know this doesn’t work … I need to think of an example that I know will work so I can give it to the students. They will go away do it and find out for themselves that it doesn’t work. Because me saying ‘no, won’t work’, they might accept it but they won’t understand why. Whereas at least if I can give them an example that contradicts their thinking.’

This is in line with the suggestion proposed by educators such as Lerman (1983), von Glaserfeld (1995) that students construct knowledge through mental ‘disequilibriums’. In other words, the teachers reconceptualized their role in the classroom as facilitators in the development of their students’ mathematical constructions rather than the sole source of mathematical knowledge.
Conclusion

This research illustrates that scaffolding is a teaching strategy that can enhance mathematics learning and help implementing constructivism in the classrooms. However, five critical features need to be addressed for successful scaffolding. These are: (1) students explain and justify their solutions, (2) teachers continuously assess students’ understanding, (3) teachers take into consideration students’ perspectives, (4) scaffolding tailor to the needs of students and (5) students take up or use the scaffolding. Finally, teachers need to reconceptualize their role as facilitators in the development of the students’ mathematical constructions rather than the sole source of mathematical knowledge while employing scaffolding in the classrooms.

References


