BENGKEL INOVASI PEDAGOGI

Mastering Basic Facts of Addition and Subtraction Through "Finger Arithmetic"

by

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ABSTRACT

"Finger arithmetic" is a strategy that uses bodily movement to represent and perform certain mathematical operations. It is used as a means to help pupils to think about the mathematics being taught to them. This workshop introduces two sets of such strategies to obtain basic facts of addition and subtraction. The first set of strategies was developed in an instructional project involving a group of Penan children in Baram. The second set of strategies was modified from a set of "finger arithmetic" strategies used in China. Workshop participants will have the opportunity to experience both sets of strategies through hands-on sessions and instructional implications will also be discussed. Since "finger arithmetic" is a link between concrete manipulative and abstract mental arithmetic, this workshop concludes that, while engaging themselves in "finger arithmetic" activities, pupils should always be encouraged to develop the ability to obtain basic facts of addition and subtraction without using their fingers.

INTRODUCTION

In many Malaysian classrooms, primary children's poor foundation in basic skills, especially skills related to the four arithmetic operations, has continually been quoted as the main reason for their poor performance in mathematics. As such, exploring effective ways to help these children to master the basic facts of the arithmetic operations has remained as one of the main centres of attention of many primary mathematics teachers. Due to Piagetian's theory of intellectual development (Orton, 1987), the use of concrete manipulatives has stayed as one major approach in this exploration.

Without doubt, the use of concrete manipulatives has assisted many children to improve their mastery of the basic facts of arithmetic operations. However, learning basic facts of arithmetic operations is more than just building the ability to recall the answers when called on. A full understanding of the meanings associated with each basic fact is essential in allowing these facts to be used in solving problems. For instance, automatic and accurate access to the addition and multiplication basic facts is not only important in itself, but also needs to form the basis for dealing with subtraction and division situations. In other words, mastery of basic facts of arithmetic operations is not just an end of children's learning of mathematics. Rather, it is also a means to acquire other conceptual understanding in mathematics. Thus, children who could free themselves from the dependency on concrete manipulatives as early as possible will find it much easier to master other higher mathematical skills. Otherwise, they will very likely find it extremely difficult to learn the mathematics being taught to them at the later stage. Hence, there

is a crucial need to develop instructional methods that will allow children to use concrete manipulatives in a systematic and meaningful manner in which they will be guided to actively construct their own meanings for arithmetic operations and to develop meaningful mental processes to perform these operations (Booker, Bond, Sparrow & Swan, 2004).

This workshop introduces two sets of finger-arithmetic strategies to obtain basic facts of addition and subtraction. The first set of strategies was developed in an instructional project involving a group of Penan children in Baram (Gan & Miehael, 2007). The second set of strategies was modified from a set of "finger arithmetic" strategies used in China (Shi Feng Shou, 1985).

"FINGER ARITHMETIC" FROM A LOCAL INSTRUCTIONAL PROJECT

"Finger arithmetic" is a strategy that uses bodily movement to represent and perform certain mathematical operations (Kagan & Kagan, 1998). Since fingers are probably the most "easily available manipulatives" in any classroom, it is no surprise that a lot of primary children are taught to use their fingers to perform arithmetic operations. However, the effectiveness of this strategy depends very much on the way it is used. Ideally, this strategy should not be treated just as a mechanical way to obtain basic facts of arithmetic operations. An instructional project carried out in an interior school in the Baram District, Sarawak (Gan & Miehael, 2007) indicates that this strategy is most effective when it is used as a means to help pupils to think about the mathematics being taught to them. A set of strategies to obtain basic facts of addition and subtraction developed in this project is introduced here. This set of strategies combines the use of ten fingers, mental images, the count-on strategy for addition, and the take-away strategy for subtraction.

Finger Representation of Numbers 1 to 10

Using fingers to represent numbers is an old tradition in almost all mathematics classrooms. However, it is important for teachers to guide pupils to use their fingers to represent numbers in a systematic and consistent way. Figure 1 shows a suggested way to use fingers to represent the numbers 1 to 10 consistently and systematically. In this system, the numbers 1 to 10 are represented by the up-fingers.

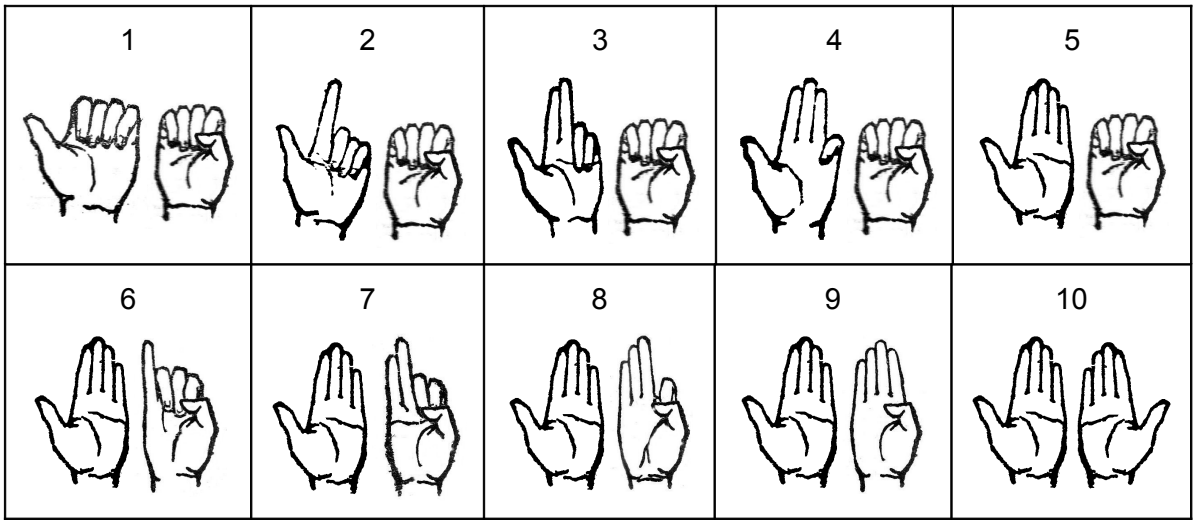

Figure 1. Representing numbers 1 to 10 by fingers

Finger Addition with the Highest Sum of 18

The sum of two single-digit numbers can be obtained easily using fingers and the count-on strategy. Basically, this strategy encourages the pupil to "put" the first number in his or her head; shows the second number with his or her fingers; and then obtains the answer by counting on from the number "put" in the head. Figure 2 illustrates the process of performing [7 + 5].

Step 1: *"7 in the head and five fingers."*

Step 2: *" ... 7, eight, nine, ten, eleven, twelve."*

Step 3: *"The answer is 12."*

Figure 2. Finger addition for [7 + 5]

It is worthwhile to note that addition of two numbers can also be obtained by the count-all strategy. For example, to obtain the sum [5 + 3], a pupil could also show both the numbers by fingers and then count all the fingers to obtain the answer. For addition with sum bigger than 10, pupils often use "tally marks" to replace their fingers in order to add the numbers using the count-all strategy. However, observation from the local project indicates that the dependency of some pupils on the count-all strategy was found to "interfere" and slow down their mastery of the count-on strategy. These pupils were often seen to resort to drawing "tally marks" when they failed to get the sum for addition more than 10 with their fingers. Since the count-on strategy is more "efficient" than the count-all strategy, it is important to avoid the count-all strategy when learning finger arithmetic. Thus, it is suggested that teachers should introduce the count-on strategy to the pupils right from the beginning of finger arithmetic.

Finger Subtraction within the Range of 10

Basic facts of subtraction within the range of 10 could be obtained easily by using ten fingers and the take-away strategy. Basically, the pupil shows the first number by his or her fingers, then "takes away" the second number by bending that number of fingers to obtain the answer. Figure 3 illustrates the process of performing [8 – 5].

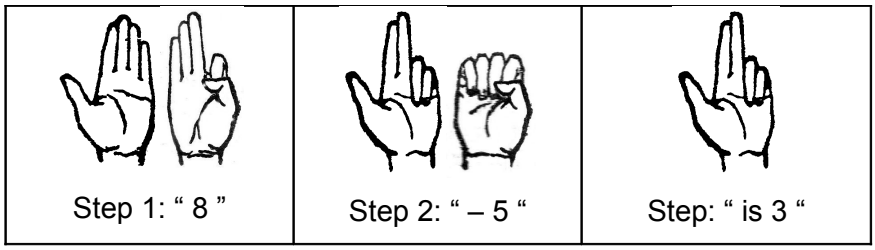

Figure 3. Fingers subtraction for [8 – 5]

Although finger arithmetic can actually be done in many different ways, it is important to guide pupils to learn in a way that is consistent with their learning of other arithmetic methods in order to promote a better understanding of the arithmetic operation. It is worthwhile to note that the strategy described in Figure 3 is consistent with the linear number-line model of subtraction as shown in Figure 4.

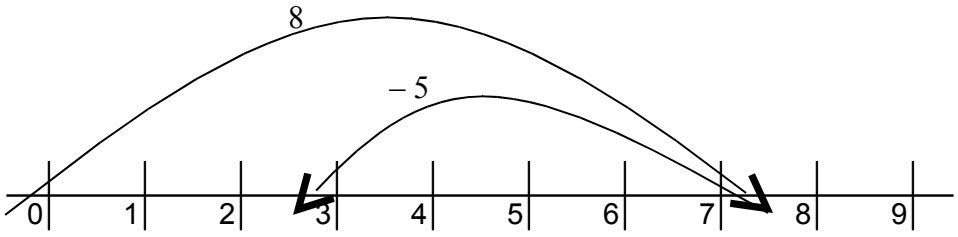

Figure 4. Number-line model of subtraction for [8 – 5]

Finger Subtraction within the Range of 18

Subtraction within the range of 18 can be performed easily using ten fingers and the count-on strategy. Basically, this strategy encourages the pupil to represent any teen number (e.g. 12) by "putting" the 2 ones in his or her head and showing the 1 ten by his or her ten fingers. The pupil will then minus the second number by "take away" that number of fingers and then add the remaining fingers with the number "put" in his or head. Figure 5 illustrates the process of performing [12 – 7].

When guiding pupils to learn this strategy, it is worthwhile to note that this method is based on the idea that 12 – 7 = (2 + 10) – 7 = 2 + (10 – 7). Furthermore, during the project, observation showed that at the early stage, many pupils made mistakes because they forgot the number "put" in their heads. Our Penan pupils taught us that touching their heads in Steps (1) and (4) was helpful in assisting them to "put in" and "take out" the number at the early stage.

Step 1: *"2 in the head."*

Step 2: *" ... and 10 fingers."*

Step 3: *" ... 10 minus 7."*

Figure 5. Finger subtraction for [12 – 7]

Step 4: *" ... 2 in the head."*

Step 5: *" ... 2, three, four, five."*

Step 6: *"The answer is 5."*

Figure 5 (cont'd). Finger subtraction for [12 – 7]

MODIFIED "FINGER ARITHMETIC" FROM CHINA

The second set of finger arithmetic strategies was modified from a set of "finger arithmetic" strategies used in China (Shi Feng Shou, 1985). In order to use this set of strategies, children first need to extend the system used to represent numbers by fingers from 11 to 20. As shown

in Figure 6, the numbers 11 to 20 are represented by one ten (mentally) and the down-fingers. It is worthwhile to note that, in learning this system of representation, it is important for children to train themselves to be able to recognise the numbers without actually counting the fingers.

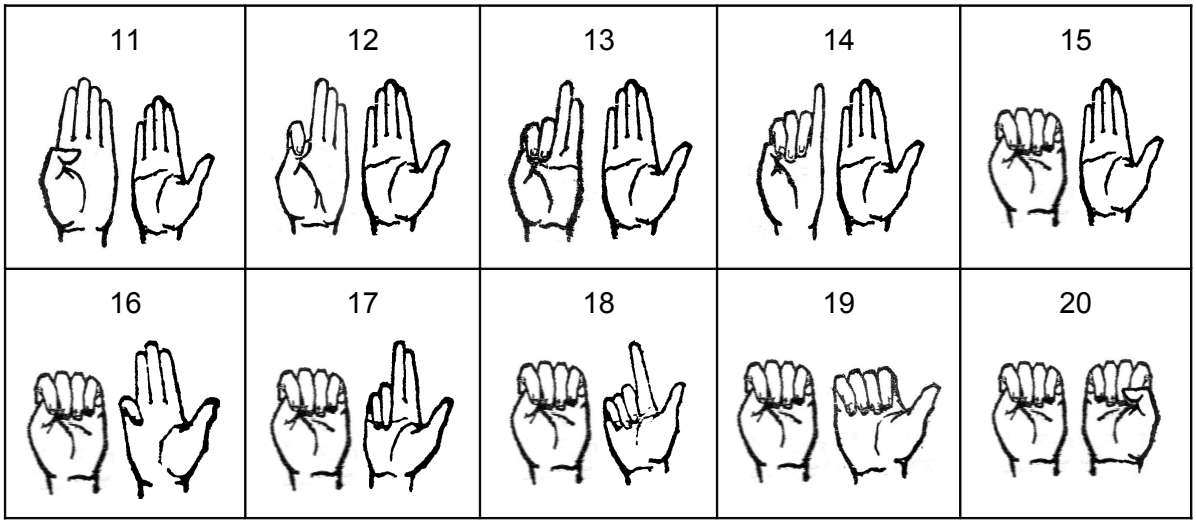

Figure 6. Representing numbers 11 to 20 by one ten mentally and the down-fingers

Addition with the Sum Up to 10

For adding two numbers with a sum 10 or less, fingers are used to add the two numbers directly. For example, to perform [3 + 4], we first show the number 3 by three up-fingers, then put up another four fingers to add 4 as shown in Figure 7. It is important to note that, a pupil who has acquired the ability to recognise the number 1 to 10 without the need of counting the fingers will be able to say the sum instantly. So, once again, the count-all strategy should be discouraged.

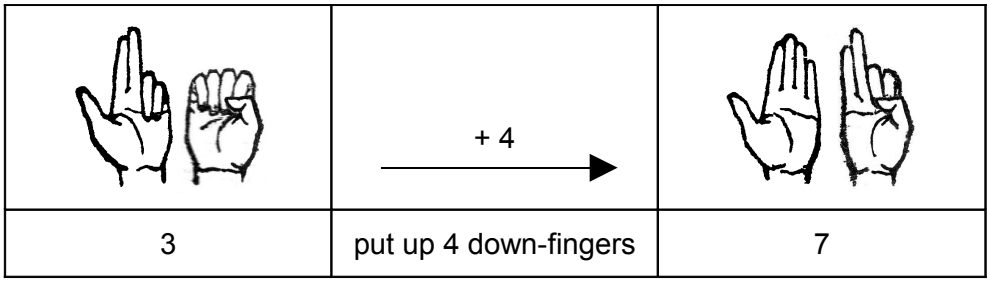

Figure 7. Finger addition for [3 + 4]

Adding 10 to a Number

Adding 10 to a number from 1 to 10 can be perform in a very easy and interesting way, by just reversing the positions of all the fingers (i.e. put up down-fingers and put down up-fingers). For example, to perform [8 + 10], we first show the number 8 by eight up-fingers, then we reverse all the fingers to get the answer 18 as shown in Figure 8.

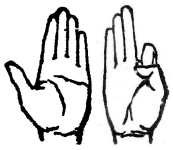	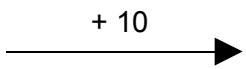	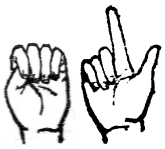
8	reverse all fingers	18

Figure 8. Finger addition for [8 + 10]

It is important to note that subtracting 10 from a number 11 to 20 can be perform in exactly the same way. Reversing all fingers from the number 18 will show the number 8, which is the answer for [18 – 10] as shown in Figure 9.

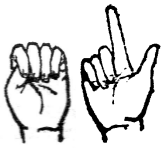	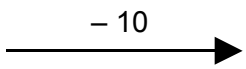	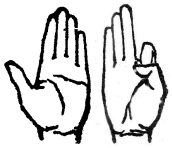
18	reverse all fingers	8

Figure 9. Finger subtraction for [18 – 10]

Addition with the Sum More Than 10

For adding two numbers with a sum more than 10, the sum could also be obtained directly by using fingers and the count-on strategy. We first show the first number by fingers, and then continue to add the second number by the remaining fingers to get the answer by counting on from the first number. Figure 10 shows the process of performing [4 + 8] using this strategy.

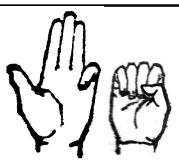	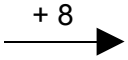	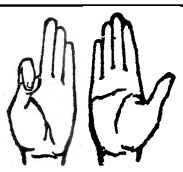
4	count-on by putting up 6 fingers and then putting down 2 fingers	12

Figure 10. Finger addition for [4 + 8] by count-on strategy

Alternatively, the sum of [4 + 8] could also be obtained by first adding 10 to the first number and then minus the complement of 10 for the second number. First, we show the number 4 by fingers, and then reverse all fingers to add 10, then put up two fingers to minus 2 (since 2 is the complement of 10 for 8) as shown in Figure 11. The mathematical process for this computation is [4 + 8 = 4 + (10 – 2) = 12].

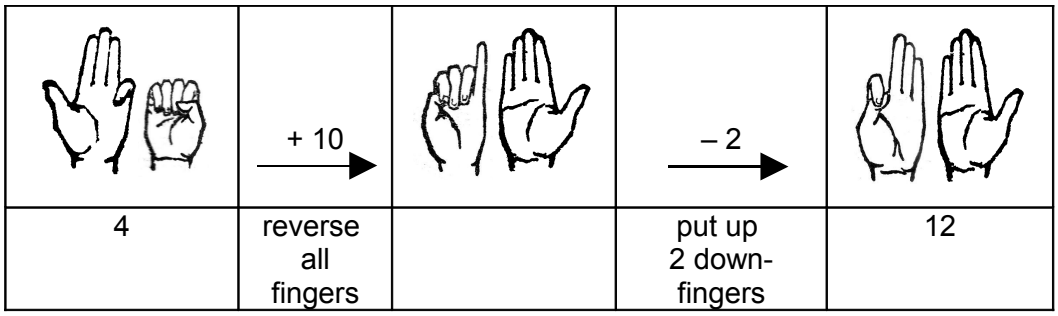

Figure 11. Finger addition for [4 + 8] by reversing-all-finger strategy

Basic Facts of Subtraction

To subtract a number from 10 or less, such as [7 – 2], the number 2 is "taken away" from 7 by putting down 2 up-fingers. The method used is the same as the strategy in the local project described before.

However, to subtract a number from a number more than 10, the take-away strategy needs to be used in a more careful manner. When the number is more than 10, "take away" is performed by putting up the down-fingers. When the number reaches 10 or less, "take away" is performed by putting down the up fingers. For example, to perform [15 – 7], we first show the number 15 by fingers, then we put up 5 down-fingers, and put down 2 up-fingers to "take away" number 7 from 15 as shown in Figure 12.

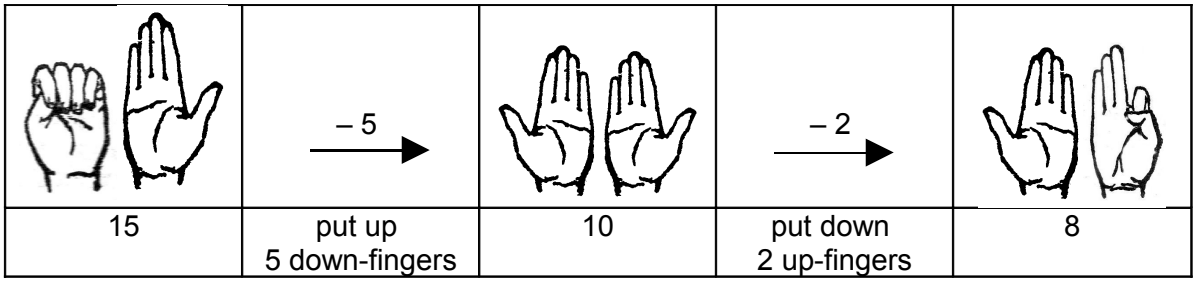

Figure 12. Finger subtraction for [15 – 7] by take-away strategy

Alternatively, the answer for [15 – 7] could also be obtained by first subtracting 10 from the first number and then adding the complement of 10 for the second number. First, we show the number 15 by fingers, and then reverse all fingers to minus 10, then put up 3 fingers to add 3 (since 3 is the complement of 10 for 7) as shown in Figure 13. The mathematical process for this computation is [15 – 7 = 15 – 10 + 3 = 8].

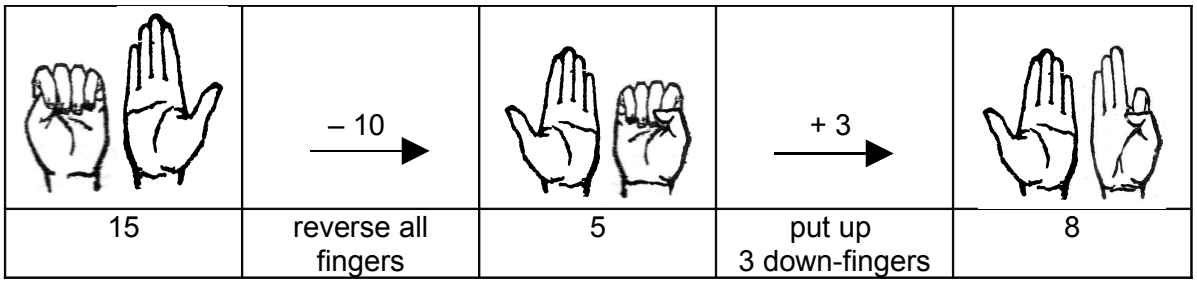

Figure 13. Finger subtraction for [15 – 7] by reversing-all-finger strategy

INSTRUCTIONAL IMPLICATIONS

To a great extend, failure in mastering basic facts of addition and subtraction has contributed to many primary school pupils' poor performance in mathematics. Hence, effective way in guiding these pupils to master the basic facts at their early stage of schooling is crucial in determining the success of their learning of mathematics in the later stage. In this respect, the potential of finger arithmetic could be explored to the fullest.

Primary pupils who are over dependent on concrete strategies such as drawing tally marks to get basic facts of addition and subtraction will inevitably find it difficult to successfully perform other higher level mathematical tasks. As such, there is a need to free them from the dependency on concrete strategy eventually. Thus, finger arithmetic should not be treated as a final end of pupils' learning. Instead, it should progress as a means to develop pupils' abilities to perform arithmetic operations automatically in their minds. In this respect, some pupils in the local project had demonstrated a good ability to obtain the basic facts without depending on their fingers at the later stage of their learning. Possibly, the use of the count-on strategy has promoted these pupils' ability to form mental images that enables them to perform the operation mentally.

To enhance the development of pupils' ability to perform the addition and subtraction mentally, they should be encouraged to recognise numbers represented by their fingers visually, without the need to actually count them one-by-one. Once they can do this, their efficiency in performing finger addition and subtraction will definitely be higher.

In the local project, finger arithmetic had shown a good potential in helping some pupils to construct their understanding of addition and subtraction. If different models of addition and subtraction are introduced to pupils, it is crucial to ensure that these models are consistent in order to facilitate formation of meaningful connection between them.

From the experience of the local project, one crucial factor in ensuring pupils' success in finger arithmetic is to ensure that every pupil has lots of opportunity to practise finger addition and subtraction. In the classroom context, sets of flash cards with a basic fact of addition or subtraction written on each of them could be prepared for pupils to learn and explore finger arithmetic in small groups. In this small-group setting, all pupils could practise finger arithmetic with one pupil holding a set of flash cards acting as the "little teacher" who will ask the questions and check the answers with reference to the flash cards.

Finally, the "reverse all fingers" method of adding 10 to and subtracting 10 from a number in the modified model from China is a very interesting feature for many young children. Hence, its potential in the Malaysian classrooms is yet to be explored to the fullest.

CONCLUSION

Observation from the local project indicates that "finger arithmetic" has a great potential in helping Malaysian young children to master basic facts of addition and subtraction. In addition, it also seems to provide a link between concrete manipulative and abstract mental arithmetic. Thus, while engaging themselves in "finger arithmetic" activities, pupils should always be encouraged to develop the ability to obtain basic facts of addition and subtraction without using their fingers.

REFERENCES

Booker, G., Bond, D., Sparrow, L. & Swan, P. (2004). *Teaching primary mathematics. 3rd ed.* NSW, Australia: Pearson Education.

Gan, T. H. & Miehael, P. P. (2007). *Fulfilling children's learning needs. How to help Penan pupils to learn Mathematics.* Miri, Sarawak: Institut Perguruan Sarawak.

Kagan, S. & Kagan, M. (1998). *Multiple intelligences. The complete MI book*. San Clemente, CA: Kagan Cooperative Learning.

Orton, A. (1987). *Learning mathematics. Issues, theory and classroom practice.* London: Cassell Education.

Shi Feng Shou. (1985). *Zhu shuan shu shuan fa – zhi shuan xing shuan yi zhu shuan xiang jie he de si zhe yun shuan.* Beijing: Nong Cun Du Wu Chu Ban She.