PROMOTING CONCEPTUAL UNDERSTANDING OF ALGEBRA USING VIRTUAL MANIPULATIVES: A STUDY OF FORM 4 STUDENTS

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Abstract

This study was conducted to explore students' perceptions of the usefulness of virtual manipulatives in promoting conceptual learning of algebra based on the constructivist learning theory (Bruner, 1996), which focused on the use of mathematical representations in conceptual understanding. In this study, virtual manipulatives was used as the instructional method in learning algebra among Form 4 secondary school students. Fifteen Form 4 students were interviewed regarding their conceptual understanding of algebra and whether vitual manipulatives promoted their conceptual understanding of algebra. Content analysis of interview data showed that the use of virtual manipulatives was useful in promoting students' conceptual understanding of algebra. In conclusion, implications of the findings and suggestions for further research are put forward.

Keywords: Virtual manipulatives, mathematics, conceptual understanding, Form 4 students

Abstrak

Kajian ini dilakukan untuk menerokai persepsi pelajar tentang penggunaan manipulatif maya dalam pembelajaran algebra secara konseptual berasaskan teori pembelajaran konstruktif (Bruner, 1996), yang berfokus kepada perwakilan matematik dalam pemahaman konseptual. Dalam kajian tersebut, manipulatif maya digunakan sebagai kaedah pengajaran dalam pembelajaran algebra dalam kalangan sekolah menengah pelajar Tingkatan 4. Seramai lima belas orang pelajar Tingkatan 4 telah ditemu bual tentang pembelajaran algebra secara konseptual dan samada manipulatif maya menambahbaikkan pemahaman konseptual mereka dalam algebra. Analisis kandungan dari data temu bual menunjukkan bahawa penggunaan manipulatif maya adalah berguna dalam meningkatkan pemahaman konseptual pelajar dalam algebra. Sebagai kesimpulan, implikasi penemuan kajian dan cadangan untuk penyelidikan selanjutnya dikemukakan.

Kata kunci: manipulatif maya, matematik, pemahaman konseptual, pelajar Tingkatan 4

Introduction

Algebra is a branch of mathematics which focuses on the manipulation of mathematical and algebraic symbols. However, these symbolic representations are abstract in nature. Hence, for most students, the learning of algebra is difficult and boring (Ignacio, 2006). Presently, the main obstacle concerning the learning of algebra in Malaysian schools is the effectiveness of mathematics instructions. The factors that affect the effectiveness of mathematics instructions are: firstly, the teacher-centred classroom instruction (Effandi Zakaria & Zanaton Iksan, 2007; Koh & Hong, 2007); secondly, the sluggishness of teachers in adopting information communication technology (ICT) in mathematics instructions (Mokhtar Nawawi, Ahmad Fauzi Ayub, Wan Zah Wan Ali, Aida Suraya Yunus, & Rohani Ahmad Tarmizi, 2005); and lastly, the influence of the examination-oriented teach-to-test culture (Lim & Hwa, 2006; Yahya Hasan, 2005). These factors have promoted the rote and procedure learning of algebra which emphasized on symbolic manipulation over concrete manipulation without developing much conceptual understanding on algebra (Effandi Zakaria & Zanaton Iksan, 2007).

Manipulatives have been recommended by the educators in the West to be used in classroom teaching (National Council of Teachers of Mathematics, 1989). For instance, physical manipulatives have long been used to facilitate students' conceptual understanding and problem solving skills (Durmus & Karakirik, 2006). However, recent advancements in ICT have opened up opportunities in promoting the use of virtual manipulatives to improve mathematics instructions (Moyer, Bolyard, & Spikell, 2002), to assist teachers in enhancing their teaching skills, and to improve the learning experience of students (Yahya Hasan, 2005). Virtual manipulatives is fast emerging as an important supplement to the present classroom instruction in improving students' learning of algebra.

Since a review of the literature shows that there is a still lack of research on virtual manipulatives in Malaysian schools, this study aimed to investigate students' perceptions of the usefulness of virtual manipulatives as a tool in promoting conceptual understanding in learning algebra. Currently, rote learning is widely practiced in the learning of algebra in Malaysian schools (Effandi Zakaria & Zanaton Iksan, 2007 reference). In addition, this study hoped to promote interest in the use of virtual manipulatives and to encourage school teachers to use virtual manipulatives as an alternative form of instruction in classroom teaching.

Purpose of the Study

This study was designed to gain further knowledge on the use of virtual manipulatives in the learning of algebra by conceptual understanding. In order to achievement this aim, this study intended to determine (a) the usefulness of virtual manipulatives in promoting students' conceptual understanding of algebra, and (b) how virtual manipulatives promoted students' conceptual understanding of algebra.

Literature Review

Conceptual understanding generally refers to students' ability to apply their knowledge on solving mathematical problems using various modes of representations of mathematical concept and to explain the solution without relying solely on restating the mathematical rules and procedure (Bolyard, 2005). Mathematics learning has been experiencing a gradual change from rote and procedures learning by symbolic manipulation to conceptual understanding by building a system of mental representations of concepts (The National Council of Teachers of Mathematics, 2000; Pusat Perkembangan Kurikulum, 2004).

The constructivist learning theory based on the work of Bruner (1966) has become increasingly popular as a guiding theory for effective learning. According to Bruner (1966), an individual can think about a particular idea or concept at three different stages: (a) enactive; (b) iconic; and (c) symbolic. The theory focuses on the way learners interact with and create meaning from their environment and experiences progressively when they move from the concrete stage to the abstract stage.

However, having the ability to move from one stage to the next as in the progressive stages was insufficient to ensure understanding of a concept. Many researchers have noted that mathematical representations can play the role in the development of conceptual understanding. Hence, for conceptual understanding to evolve, students need to be able to perform two additional tasks: (a) translation between multiple representations of a concept; and (b) interaction between internal representation and external representation of a concept.

Lesh, Landau, and Hamilton (1983) identified five distinct types of representation systems (Figure 1) that occur in mathematics learning and problem solving. They emphasized the importance of connection and translation among algebra representations to solve problems.

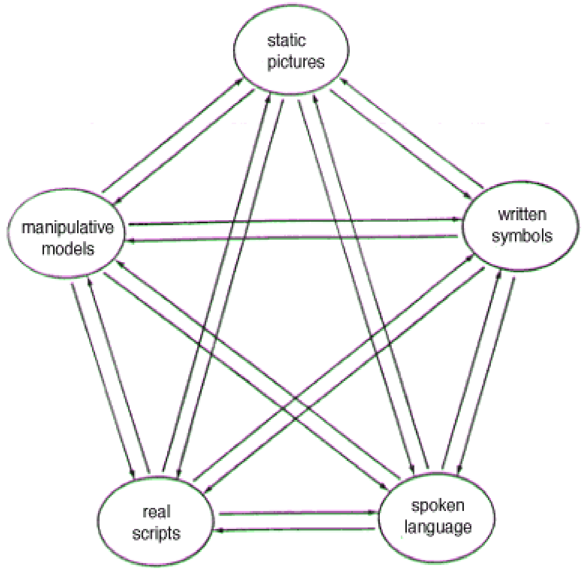

Figure 1. Five Representation Systems in Mathematics Learning

Reys, Lindquist, Lambdin, and Smith (2007) suggested that students should experience many different representations of mathematical concepts. Only after the students have been exposed to different external representations like words and symbols representation of a concept, would they be able to internalize the concept when they achieve the stage of interaction between internal and external representation (Figure 2).

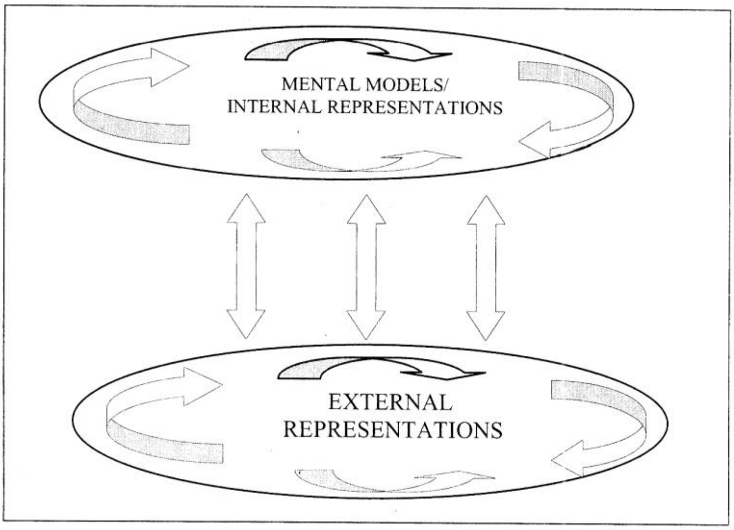

Figure 2. Interactions between Learners' Internal and External Representations

The above reviews have demonstrated how mathematical concepts can develop through: first, moving progressively from the concrete stage to the abstract stage in mathematics learning; secondly, encouraging the student to be actively involved in the use of multiple representations in mathematics; and thirdly, emphasizing on connections and translations among different mathematical representations.

Virtual manipulatives refer to the dynamic visual representations of concrete manipulatives (Spicer, 2000). They are computer software programmes which emulate physical manipulatives by keyboard operation instead of physical action on the three-dimensional objects. Previous studies showed that students who worked with virtual manipulatives demonstrated higher achievement and better attitudes toward mathematics (Namukasa, Stanley, & Tuchtie, 2009), significant improvement of students' conceptual knowledge in fractions (Reimer & Moyer, 2005), and students using virtual manipulatives outperformed students using physical manipulatives in learning fractions (Suh, 2005). Hence, the present study aimed to explore students' perceptions of the usefulness of virtual manipulatives in conceptual learning of algebra.

Method

Participants

This study was carried out in three secondary schools in Kuching. A total of 207 Form Four students participated in this study. Due to the time constraint, only fifteen of them were selected for interview. The selection for interview was not random but conveniently selected to have a balanced representation from all the schools.

Instrumentation

This research used virtual manipulatives as the method of instruction. The virtual manipulatives (Figure 3) used in the study were developed based on the guidelines and recommendations from past studies such as Moyer et al. (2002), and Durmus and Karakirik (2006).

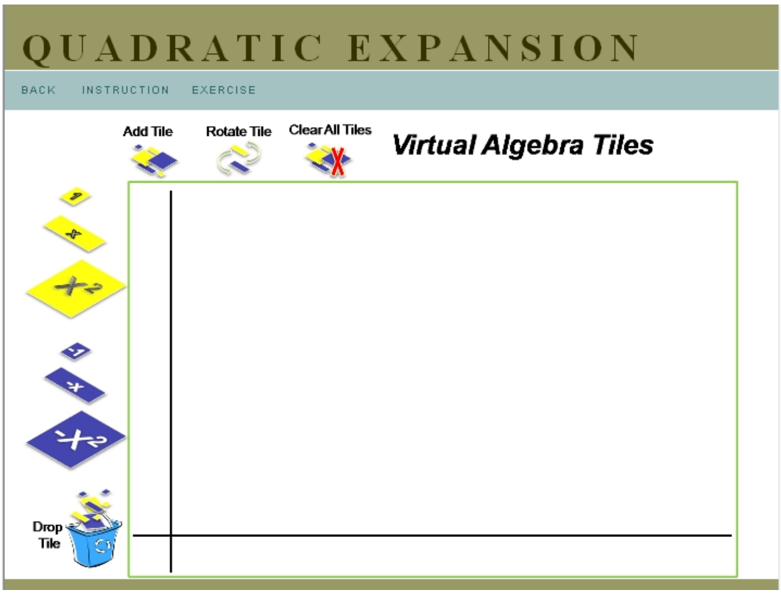

Figure 3. The Virtual Manipulatives used in the Study

A conceptual understanding task sheet on quadratic expansion was used to facilitate the collection of qualitative data. The conceptual task sheet consisted of three items respectively. Each item required the students to present the solution to the problem in three different representational forms: (a) pictorial form – drawing algebraic tiles, (b) table form – using algebraic symbol table, and (c) written symbols form – using algebraic symbols. The items were developed based on the systems of representations and the development of mathematical concepts literature (Goldin & Shteingold, 2001) as shown in Figure 4.

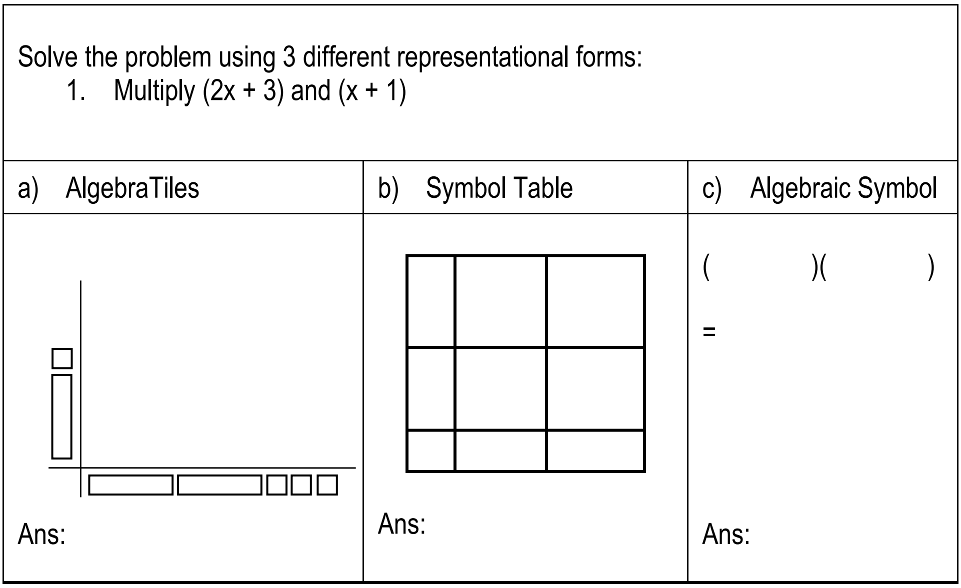

Figure 4. Example of the Item in Conceptual Understanding Task Sheet

Due to time constraint, pilot study was not done on the reliability of the conceptual understanding task sheet. However, six individuals who were identified as expert reviewers based on their background and knowledge of mathematics education conducted an analysis on the items. The results from the expert review indicated that the test items were at moderate complexity, which fulfilled the required level of difficulty in measuring students' algebraic concept and can be used in this study.

Procedure

The learning of mathematics by conceptual understanding emphasized on progressive learning from the concrete stage to the abstract stage. Hence, during the one week learning session, the students were taught how to a) make use of the virtual algebra tiles to represent / to convert the abstract symbols into concrete tiles, b) manipulate with the tiles to solve some simple problems and c) convert back the concrete tiles to the abstract symbols. At the end of the session, they were asked to complete the conceptual task sheet as mentioned above and followed by standardized open-ended interviews

(Patton, 2002). The questions prepared for the interviews were of types like experience, knowledge and feeling base on their works on conceptual task sheet with follow up questions and probes like detailed oriented questions, elaboration questions and clarification questions (Patton, 2002). The interviews took place face-to-face and each interview was audio-taped and transcribed verbatim.

Data Analysis

The collected data were analysed using the constant comparative method (Maykut & Morehouse, 1994). The analysis began by identifying the units of meaning from the interview transcripts. The units of meaning were then grouped into categories where the rules of inclusion for each category as well as the examples were prepared and listed down as in Table 1.

Table 1

Example on the Coding of the Students' Interview Transcript

Category	Rule of inclusion	Example
Novelty effect.	Students found the algebra tiles to be a new experience, which could motivated the students toward learning algebra.	“The algebra tiles is easy to use and I would like to share my experience with my friend in the other class.”

After preparing the lists of categories, rules of inclusion, and examples, the researcher coded all the interview transcripts.

Findings

The interview responses of the students were content analysed and the findings are as follow:

(a) The usefulness of virtual manipulatives in promoting students' conceptual understanding of algebra

In analyzing the interview transcripts, first, the researcher was interested to know whether virtual manipulatives was useful on students' conceptual understanding of algebra. The analysis of the students' work revealed that the use of virtual manipulatives has created a new learning experience that might motivate the students by drawing their interest to learn algebra. This consistent theme was the novelty effect of using the virtual manipulatives as the instructional tool. A student commented: “I have a good friend in the other class and I feel that it is good to let him know the other way of doing quadratic expansion. I am sure he will enjoy it [virtual manipulatives]” (Student 8).

The analysis also revealed that students that prefer solving problems using algebra tiles benefited from virtual manipulatives. Another consistent theme was preferences over virtual manipulatives. A student’s comment that showed the preferences was: “I like using algebra tiles. It is simple and easy to understanding, especially when dealing with negative tiles. Algebraic symbol is difficult when come to negative sign.” (Student 11)

(b) How virtual manipulatives promoted students’ conceptual understanding of algebra

Next, the researcher was interested in how students explained their mathematical thinking and justified their work. This would contribute to the main research interest which focused on conceptual understanding of algebra. Hence, the researcher was interested to know the ability of the students in using mathematical representations, that is, whether the students used various representational forms to explain their reasoning or they only relied on stating rules and procedures when solving the problem.

The analysis of the students’ work revealed that there were evidences to show that students using virtual manipulatives were able to work by using various representational forms (pictures, tables, and symbols). One theme was that the virtual manipulatives was helping the students to explain the solution to the problem using various representations of concept. An example of this theme was commented as: “After I have done with the drawing, I change all the tiles [algebraic tiles] to symbols [algebraic terms]. Then I put all the symbols [algebraic terms] in each of the four boxes in the symbol table. Then I add up all the four symbols [algebraic terms] inside the symbol table. So I get the answer” (Student 9).

The analysis further revealed that students showed confidence in solving the problem. Another consistent theme was that students showed an improvement in algebraic skill by using one or more representational forms to counter check their work. A student's comment that reflected the improvement was: "I have to wait 3 times, first algebraic symbol, then check with symbol table, and finally I check again with algebraic tiles, and I got the same answers" (Student 4).

Discussion

In this study, initial findings revealed that virtual manipulatives was useful as an alternative instructional tool that provide new learning experience to the students and has drawn their attention in the interest of learning algebra. The findings also revealed that students were able to solve the quadratic expansion problems as well as to facilitate, explain and counter check/self evaluate their work using various representational forms. This was in accord with the report by Moyer, Niezgoda, and Stanley (2005) that students were able to represent their thinking in various representational forms using virtual manipulatives. This ability to move flexibly between different representations of a concept indicated conceptual understanding of that concept (Lesh, Post, & Behr, 1987).

Conclusion

Summary

This study investigated the use of virtual manipulatives that was fast emerging in the literature as an alternative classroom instruction to facilitate the conceptual development of the students. The findings from this study provided some positive results on the use of virtual manipulatives in learning algebra. The usefulness of virtual manipulatives was demonstrated by the novelty effect in using a new instructional tool that most of the students have never seen before. Likewise, the conceptual learning of algebra using virtual manipulatives has shown improvement in algebra and was indicated clearly by students' confidence in solving the problems using multiple representations of the concept. However, virtual manipulatives should not make the traditional classroom instruction obsolete but instead be used as a supplementary instructional tool to enhance the classroom teaching and learning. Although the use of virtual manipulatives has been enhanced with progress in ICT, more research is needed to fully understand the various issues surrounding virtual manipulatives instruction. The findings of this study contribute to the body of research exploring this question.

Implications

Two major contributions can be drawn from this study. In term of theoretical contribution, the results of this study supported the concrete to abstract learning theory by Bruner (1966) and the work of Lesh et al. (1983) which showed that manipulatives could be used to promote the conceptual understanding of algebra. Likewise, the study also confirmed that the ability to translate between different mathematical representations of a concept was the key to learning by conceptual understanding. This result was consistent with the work by Reys et al. (2007). In term of practical contribution, although Domino's (2010) study reported that manipulatives has positive effect in primary school on mathematics achievement, the finding in this study found that virtual manipulatives was also effective in secondary school as well. Hence, teachers should encourage and motivate students to explore new algebra concepts by focusing more on conceptual learning using virtual manipulatives rather than on rote and procedure learning of algebra in class. The findings of this study could be used as references by the students and the teachers in promoting the interest in the learning of algebra.

Suggestions for Further Research

Future research can be carried out base on the results of this study by focusing on a) longer time for treatment and follow up studies on students' retention skills, b) replicating the study using larger sample involving more schools and students from other forms beside form four classes, and c) exploring different mathematics contents beside algebra like geometry and trigonometry.

References

Bolyard, J. J. (2005). *A comparison of the impact of two virtual manipulatives on student achievement and conceptual understanding of integer addition and subtraction.* Unpublished doctoral dissertation, George Mason University, Fairfax, Virginia, U.S.A.

Bruner, J. S. (1966). *Toward a theory of instruction.* Cambridge: Harvard University Press.

Domino, J. (2010). *The effects of physical manipulatives on achievement in mathematics in grade K-6: A meta-analysis.* Unpublished doctoral dissertation, State University of New York, New York, U.S.A.

Durmus, S., & Karakirik, E. (2006). Virtual manipulatives in mathematics education: A theoritical framework. *The Turkish Journal of Educational Technology, 5*(1), 117-123.

Effandi Zakaria, & Zanaton Iksan (2007). Promoting cooperative learning in science and mathematics education: A Malaysian perspective. *Eurasia Journal of Mathematics, Science & Technology Education, 3*(1), 35-39.

Goldin, G., & Shteingold, N. (2001). Systems of representations and the development of mathematical concepts. In A. A Cuoco & F. R. Curcio, (Eds.)*, The roles of representation in school mathematics NCTM yearbook 2001* (pp. 1-23)*.* Reston, VA: NCTM.

Ignacio, N. G. (2006). The affective domain in mathematics learning. *International Journal of Mathematics Education, 1*(1), 16-32.

Koh, C. K., & Hong, K. S. (2007). Teachers' attitudes toward computers: An overview of four studies done in Sarawak. In K. S. Hong, Napsiah Mahfoz, T. L. Liau, & Abang Ekhsan Abang Othman (Eds.), *Contemporary issues in education* (pp. 11-27). Kuching, Sarawak: Persatuan Perkembangan Profesionalisme Pendidikan Sarawak.

Lesh, R., Landau, M., & Hamilton, E. (1983). Conceptual models in applied mathematical problem solving research. In R. Lesh & M. Landau (Eds.) *Acquisition of Mathematics Concepts and Processes* (pp. 263-343). New York: Academic Press.

Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier, *Problems of representation in the teaching and learning of mathematics* (pp. 33-40). Hillsdale, NJ: Erlbaum.

Lim, C. S., & Hwa, T. Y. (2006). *Promoting mathematical thinking in the Malaysian classroom: Issues and challenges.* Paper presented at the APEC-Tsukuba International Conference: Innovative teaching mathematics through lesson study (II) – Focus on mathematical thinking (Tsukuba University, Tokyo, Japan, December 2-7, 2006). Retrieved March 27, 2011 from: http://www.criced.tsukuba.ac.jp/math/apec/apec2007/paper_pdf/Lim%20Chap%20Sam.pdf.

Maykut, P., & Morehouse, R. (1994). *Beginning qualitative research: A philosophic and practical guide.* London: The Falmer Press.

Mokhtar Nawawi, Ahmad Fauzi Ayub, Wan Zah Wan Ali, Aida Suraya Yunus, & Rohani Ahmad Tarmizi (2005). Teachers' perceptions on the conditions facilitating the use of computers in teaching mathematics. *Malaysian Online Journal of Instructional Technology, 2*(3), 88-98.

Moyer, P. S., Bolyard, J. J., & Spikell, M. A. (2002). What are virtual manipulatives? *Teaching Children Mathematics, 8*(6), 372-377.

Moyer, P. S., Niezgoda, D., & Stanley, J. (2005). Young children's use of virtual manipulatives and other forms of representation. In B. Masalski, *NCTM 2005 yearbook: Technology supported mathematics environments* (pp. 17-34). Reston VA: National Council of Teachers of Mathematics.

Namukasa, I., Stanley, D., & Tuchtie, M. (2009). Virtual manipulative material in secondary mathematics: A theoretical discussion. *Journal of Computers in Mathematics and Science Teaching, 28*(3), 277-307.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: NCTM.

National Council of Teachers of Mathematics (2000). *Principals and standards for school mathematics.* Reston, VA: NCTM.

Patton, M. Q. (2002). *Qualitative research and evaluation methods.* London: Sage Publications Ltd.

Pusat Perkembangan Kurikulum (2004). *Integrated curriculum for secondary schools: Curriculum specifications mathematics form 4.* Kuala Lumpur: Ministry of Education Malaysia.

Reimer, K., & Moyer, P. S. (2005). Third-graders learn about fractions using virtual manipulatives: A classroom study. *Journal of Computers in Mathematics and Science Teaching, 42*(1), 5-25.

Reys, R., Lindquist, M., Lambdin, D., & Smith, N. (2007). *Helping children learn mathematics* (8th ed.). Hoboken, NJ: John Wiley & Sons, Inc.

Spicer, J. (2000). Virtual manipulatives: A new tool for hands-on math. *ENC Focus, 7*(4), 14-15.

Suh, J. M. (2005). *Third graders' mathematics achievement and representation preference using virtual and physical manipulatives for adding fractions and balancing equations.* Unpublished doctoral dissertation, George Mason University, Fairfax, Virginia, U.S.A.

Yahya Hasan (2005). Mathematics education and technology: Past, present and future - The Malaysia experience. *The Electronic Journal of Mathematics and Technology, 1*(1), 90-94.