STORY PROBLEM IN MATHEMATICS (DIVISION WITH REMAINDER)

By

Christina Agang
SK Bedup,
94700 Serian, Sarawak.

Abstract

This study was prompted by my concern over my pupils' tendencies to view Mathematics as an isolated subject learned only in classroom rather than as a tool to help them in their daily lives. This study was done to see whether exposing pupils to story problems before the teaching of content would help to reduce the gap between theoretical mathematics and its application.

INTRODUCTION

I was in my third year teaching Mathematics to Class X. I came to realize that my pupils tend to view Mathematics as an isolated knowledge rather than a tool to help them understand the world better. As their teacher, I felt responsible to keep "my eye on the mathematics and my ear to the students" (Ball, 1993) (as quoted by Ball & Wilson, 1996). I worried about my pupils' mathematical understanding and about what they were learning about themselves (Ball & Wilson). Thus, I decided to pause and reflect on my teaching. And in my quest, I found many likewise worries from my fellow educators, such as, by Pengelly (1988) who stated these. *By and large, they grow up believing quite firmly that school mathematics is something done in school that has no relationship with the rest of the world, and is certainly not anything needed for the rest of their lives* (pp 4-5) (as cited by Bickmore-Brand, 1993).

Incidentally, I discovered a literature by Kamii (1990) who suggested to teachers of Mathematics to apply story problem in their teaching. Well, this is no news to Mathematics teachers as including story problem in teaching Mathematics is a normal practice. Even the activity books provided by the Ministry of Education of Malaysia (Kementerian Pelajaran Malaysia)(KPM) are rich with 'story problems' which are usually included at the end of the topics. Nevertheless, besides urging teachers to use story problem, Kamii also suggested teachers to reverse this sequence. According to him, story problem should be taught **before** the teaching of content. For Kamii,

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> "…in traditional mathematics instruction, 'story problems' are often considered as applications of computational skills and are presented after computational exercises. This sequence should be reversed because children construct mathematical knowledge out of daily living. Computation with numbers, which do not involve contexts, should come after a great deal of problem solving with real life context."

He further stressed that,

> "Mathematical symbols belong to social knowledge, that is, knowledge based on conventions worked out by people. Children who can think, that is, those who can do the logico-mathematical part of arithmetic, can easily learn to write mathematical symbols. By prematurely focusing on children's learning of symbols and by overlooking the importance of thinking (i.e., constructive abstraction), traditional mathematics instruction teaches children to manipulate symbols on paper instead of constructing logico-mathematical knowledge."

His opinion was also supported by Pengelly (1998) who stated, t*he manner in which children build mathematical knowledge is not the same as the syllabus sequence for teaching each component* (as cited by Bickmore-Brand, 1993). I was intrigued by this. I was of hope Kamii's (1990) suggestion would be the missing link in my teaching. I hope that exposing my pupils to 'story problems' before the teaching of content would bridge my pupils' understanding between Mathematics as the subject and as the tool.

PURPOSE

As I have stated earlier, I was worried that my pupils looked at Mathematics as a subject they learn in class only and that it is not related to their daily life and hence, have no use to them outside the classroom. Thus, the purpose of this study was to use 'story problem' (which are related to daily life) prior to the teaching of content in the hope to bridge the mentioned gap. The topic I picked relates to "division with remainder."

During the study, I exposed my pupils to many 'story problems' first before teaching them the long division. To measure their understanding, I carried out interviews and gave them 'story problems' throughout and at the end of the research. Hence, I deem the usage of long division and vertical form as using mathematics (as a tool) and 'story problem' (as situation) from their daily life.

LITERATURE REVIEW

On Problem Solving

I initially found it difficult to look for the definition of 'story problem' as stated by Kamii (1990). Upon probing further, Kamii seemed to easily interchanged between the usage of 'story problem' and 'problem solving' in his literature as if he was referring to the same thing (or method/ approach). For example, "*in traditional mathematics instruction, **'story problems'** are often considered as applications of computational skills and are presented after computational exercises…"* (Kamii) whilst in the same paragraph he stated, "c*omputation with numbers, which do not involve contexts, should come after a great deal of '**problem solving'** with real life context."* Initially, I decided to use the term 'problem solving' in my research. But when I looked for the definition in other literature, I found numerous definitions that were too overwhelming. The following are some of the definitions.

Kantowski (1977) stated problem solving as "*a situation for which the individual confronting it has no readily accessible algorithm that will guarantee a solution*" (as cited by Jones, 2000) where as Schoenfeld (1985) said, "*Indeed, figuring it out is what mathematics is all about* (as cited by Jones).

While Jones (2000) himself described problem solving as "*a way of thinking, of analyzing a situation, of using reasoning skills not learned through memorization of specific facts, but by immersing one-self in the problem solving process and applying both past experience and knowledge at hand",* Dewey (1910) (as cited by Lau, 2004) defined problem solving as "*a felt difficulty."* And Lau in the same literature quoted Browell (1942) who defined 'problem solving' as below.

> "Problem solving refers (a) only to perceptual and conceptual tasks, (b) the nature of which the subject by reason of original nature, of previous learning or of organization of the task, is able to understand, but (c) for which at the time he knows no direct means of satisfaction. (d) The subject experiences perplexity in the problem situation, but he does not experience utter confusion … problem solving becomes the process by which the subject extricates himself from his problem." (p 416)

I do not need to *confront my pupils with problems that they have no readily accessible algorithm that will guarantee a solution.* Neither do I want my pupils to over analyze a situation (I love the way Dewey defined it but that's neither here nor there). I only intended to use 'problem solving' as a tool to represent mathematical ideas. I merely use problem solving to pose problems from daily life at the beginning of my lessons as a tool for me to teach. Hence, in this research, the term 'story problem' seem more appropriate.

Seeking further, I discovered 'Routine Procedures in Developing Competence' by Lesh, Posh and Behr (1987). Lesh, Posh and Behr stated that *"there are five common ways in which we represent mathematical ideas: spoken language, concrete objects, pictures, real life situations and written* *symbols*" (as cited by Hiebert, 1990). Hiebert further stated that *"all the representations are useful. The context and purpose determines which representation is most appropriate."* I needed my 'story problem' (real life situations) to help my pupils build meaning in their learning of mathematics. And to aid them in their problem solving process, I have used concrete objects (hands on materials).

In 'Implications of Constructivism for Teaching Beginning Arithmetic', Kamii (1990) stated "*children construct their own logico-mathematical knowledge from the inside, we encourage them to do their own thinking and exchange viewpoints among themselves. Young children will eventually construct the algorithms that are now prematurely imposed on them*." Thus, in this study, I have tried to promote group work to allow for discussion as much as possible. Classroom discussions were also encouraged.

Besides that, Hiebert (1990) said, "*meaning or understanding in mathematics comes from building or recognizing relationships either between representations or within representations"*. Therefore, in this study, I tried to present to my pupils many situations where they would be able to construct relationships among at least three forms of representation; real-life situation (problems solving), concrete objects (through hands on activities) and written symbol (examples, x, ÷). These, I hope would help to develop meaning for the topic I was teaching.

On the limitations

As I planned my research, I could not neglect the fact that presenting my pupils with real life situation would require me to string words into sentences. This put me in a dilemma as Mathematics in Malaysia is taught in English. Unfortunately, none of my pupils are conversant in the language. I do not want the language barrier to hinder our lesson. As Clement (1980) and Watson (1980) stated, *"error based on miscue analysis in reading had included errors in reading comprehension, process skills and encoding"* (as cited by Kameenui & Jitendra, 1996) while errors derived from information processing theory have included language difficulties, spatial representations, inadequate knowledge of prerequisite concept and skills, incorrect associations, and application of irrelevant strategies (Radatz, 1979) (as cited by Kameenui & Jitendra). Nonetheless, Kameenui and Jitendra also stated that, "*however, students had more difficulty solving one-step word problems that contained extraneous information than they did simple one-step word problem."*

Hence, to avoid circumstances where my students were not able to comprehend the 'story problems' not because they have inadequate knowledge but because they simply cannot understand the question, I decided on using only simple one-step word problem.

METHODOLOGY

This study was done to fulfill the requirement of Advance Action Research Course organized by IPBL. Thus, action research is adopted here. Other than that, it is well-suited in exploring the many perspectives of the researcher and the study. And the method adopted in this research is qualitative.

The participants were my Year 3 pupils - 14 of them in total. They were of different abilities and their performances varied from one another. Two of them were high achievers that needed little coaching in their learning. Meanwhile, another three pupils, Florence, Fiona and Jack, are currently attending special remedial class where as Lina, Ann and Angel needed a lot of help in their work. George with moderate ability was a frequent absentee. The rest were moderate pupils.

In action research, a triangulation of methods is recommended to ensure that the study is trustworthy and stands up to the rigor of scholarship (Mills, 2003 as cited by Marcellino). Journals, evaluative surveys (interviews, exercises) and daily lesson plan became the data sources and provided the chronology for analysis (Marcellino, 2004). I tried to get another teacher to observe my class during my lessons and to analyze my data but time limit and distance of the study site hindered the plan.

According to Mills (2003) (as quoted by Marcellino, 2004), personal inventories and questionnaires may be included in action research. Thus, selected pupils were interviewed after the study was carried out to determine their understanding and perspectives toward division with remainder. However, no transcripts of the interviews were taken, as I translated the data (my questions and their answers) to questions pertaining to divisions.

I also referred to the pupils' exercises, test questions and my observation of their responses in class (which was written in my journal) and used them as my source of raw data. Many exercises were also given out during this study. But overall, I was only interested on my pupils' ability to answer 'story problems' related to their daily life by using the vertical form and long division taught in class.

FINDINGS AND DISCUSSION

Part 1: The Teaching of Concept with Story Problem

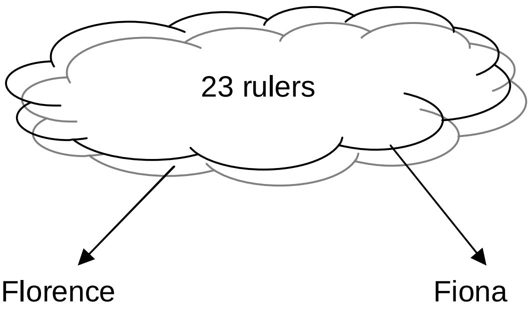
Extract from my journal and daily lesson plan showed that on April 12th, 2007, I taught my pupils how to divide 2-digit numbers by 1-digit numbers with remainders (Skill 1.3, YLP, Portal Utusan 2007). I was teaching them division with remainders for the first time. They knew how to divide one-digit number without remainder (eg.: 8÷2 = 4) and group up to 2-digit numbers to 1-digit groups (eg.: grouping 10 items to 2 groups). Nevertheless, they had no prior knowledge on dividing any number with remainders or dividing more than a single digit number using the conventional symbol representation (vertical form or long division). However, they knew the meaning of the symbol "÷".

I started the lesson by grouping them into two groups (boys and girls) before proceeding to give each group a problem. The group then strived to solve the problem. Because of their competitive nature, the problem solving part became a battle of the sexes.

The question was:

> *I have 23 rulers. I want to divide the rulers equally between Florence and Fiona. How many rulers will each of them get?*

Then I drew the diagram below on the board:

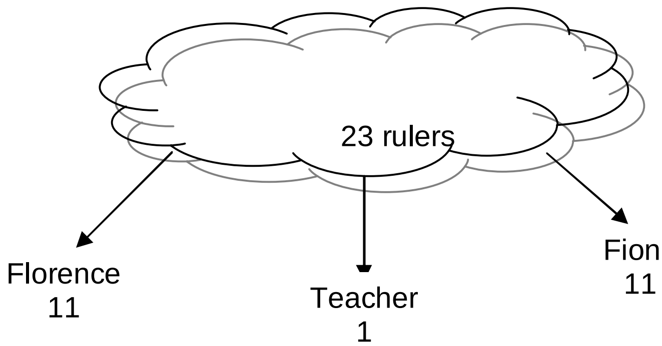


I gave each group 23 rulers (provided free by a company) to aid them in their problem solving process. The discussion became heated when everybody seem to come to the conclusion that both Florence and Fiona would get 11 rulers each. But that would leave out 1 extra ruler. The boys suggested for the extra ruler to be given to Florence. However, I reminded them of the definition of the word 'equally' and that their solution would not be fair to Fiona. They spent a few more minutes arguing among themselves before the boys decided to remove (or their exact word was; 'throw') one of the ruler. When the girls heard the suggestion, they agreed that it was the only possible conclusion.

In the heat of the matter, I asked two girls to come to the front. I divided 23 rulers between them while chanting loudly, *"One is for Florence, one is for*

Fiona, one is for Florence….” When I came to the odd ruler, I asked the class, *”What should I do with this ruler?”* Jeff answered, *“Give to teacher.”* And his answer cemented their understanding on division with remainders. Upon reflection on action, I stated this, “*Unfortunately none of them thought of dividing the one ruler to two. May be because they thought the ruler will be of no use once broken to two.”*

After that, I reminded them of the diagram written on the board. I added:



Then, I asked the class for the number sentence. They responded *“Twenty three divided by two equal 11.”* I wrote them on the board as:

23 ÷ 2 = 11

Then I explained that the one ruler we have decided to give to the teacher is called *‘remainder’*. Then I added,

23 ÷ 2 = 11 remainder 1

I explained further that, *“We can also simplify ‘remainder’ to…”* I modified my number sentence to,

23 ÷ 2 = 11 ® 1

We kept on doing likewise questions (I wrote the diagram on board, and they answered in unison) until the end of the period. At the end of the lesson, I stated in my journal, “*I felt most of the students* (sic*) were able to grasp the basic concept of division with remainder. Even F and f (my two weakest pupils) would know what to do if ask to divide 5 sweets equally between the two of them. Even they will know that they will have to give the odd sweet to me (teacher).”*

Part 2: Symbol Representation

Day 1

I taught them to do division using the vertical form and long division during the next few lessons. The pupils had been exposed to the vertical form, but only for question like, 8÷4= or 16÷2= where they can directly write down the answers as $\begin{array}{r} 2 \\ 4\overline{)8} \end{array}$ and $\begin{array}{r} 8 \\ 2\overline{)16} \end{array}$. As I reflected the day before, I believed that most of the pupils by now, had mastered the concept of division. Hence, I thought teaching the majority the vertical form of division will be easy. As (Kamii, 1990) stated that, “*those who can do the logico-mathematical part of arithmetic, can easily learn to write mathematical symbols.”*

However, the reality was just the opposite. Everybody found doing long division hard. They were confused with the proper placement of numbers.

When I wrote the previous question in vertical form, $2\overline{)23}$ they were still able to grasp the representation. But when I proceed to the long division,

$$\begin{array}{r} 11 \\ 2\overline{)23} \\ 2 \\ 03 \\ 02 \\ 01 \end{array}$$

none of them were able to follow me anymore. They had the tendency to put the answer under the numerator such as shown below,

$\begin{array}{r} 2\overline{)23} \\ 11 \end{array}$ instead of $\begin{array}{r} 11 \\ 2\overline{)23} \end{array}$

So I resorted to having them copy my whole calculation step by step. I started by dividing simple 2-digit number that could be divided with 2 or 3 on both digit without remainder. Example, 24÷2= and 36÷3= . I solved the problem step by step and everybody copied step by step. I would pause when I finished dividing the first digit to allow the pupils to copy before I proceed to divide the next digit.

Step 1

$$\begin{array}{r} 1- \\ 2\overline{)24} \\ 2 \\ -0 \end{array}$$

Step 2

$$\begin{array}{r} 12 \\ 2\overline{)24} \\ 2\downarrow \\ 04 \\ 04 \end{array}$$

We did this a few times. Nevertheless, I always started each question with a word problem similar to the 'rulers' problem. At the end of the lesson, I gave them 5 questions for homework.

Day 2

However, when I checked their exercise books the next day, none of the pupils were able to answer their homework correctly. I had to come to each of the pupil to assist them individually. Thus, prompting me to write another 5 questions on the board.

(1) 33÷3=
(2) 69÷3=
(3) 84÷4=
(4) 24÷2=
(5) 96÷3=

We solved each problem together. The pupils gave input on what number should be written and the placements of the numbers. I also asked them to copy each question and the calculation step by step, pausing every once in a while to give them time to copy. I emphasized on this step,

$$\begin{array}{r} 23 \\ 3\overline{)69} \\ 6\downarrow \\ 09 \\ 09 \\ 00 \\ \hline \end{array} \quad \Leftarrow \textit{I emphasized on this step}$$

I gave the pupils 5 more questions to do as homework.

Day 3
The next day, I started my class with a word problem concerning division of 2-digit numbers by 1-digit numbers without remainders which I translated to diagram. We solved the problem together through class discussion. My role was to write the correct responses on board only. When I found out that most of them were able to give me correct input verbally, I gave them a question to be copied and answered correctly before going around the class to check on their homework. I found out that Cas, Suzie, Flora, Jeff, Andy and Mark were able to answer their homework correctly. George was not present the day before. Frankie did not even bother to copy the questions. Ann, Angel, Lina, Florence, Fiona and Jack were not able to answer their question correctly. Florence and Fiona did not even copy the question correctly. Despite the sad statistic (only 46 percent (%) mastered the first step), I decided to move on to the next level.

Day 4
At this level, I gave them a word problem concerning division of a 2-digit number by 1-digit number with a remainder on the first digit.

Question: *I have 78 ice-creams. I want to give the ice-creams equally between Abu and Ali. How many ice-creams will each of them get?*

$$\begin{array}{l} \quad\ \ 39 \\ 2\overline{)78} \\ \quad 6\downarrow \\ \quad 18\ldots\Leftarrow \\ \quad 18 \\ \quad 00 \end{array}$$

I emphasized on this step

I also gave them questions which required them to multiply the denominator with 0.

Question: *I have 14 ice-creams. I want to give the ice-creams equally between Abu and Ali. How many ice-creams will each of them get?*

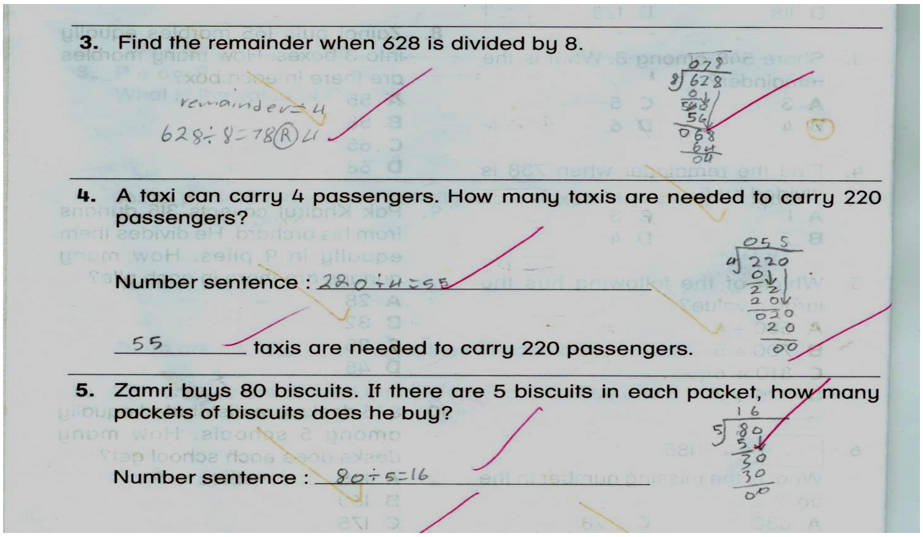
$$\begin{array}{l} \quad 07 \\ 2\overline{)14} \\ \quad 0\downarrow \\ \quad 14 \\ \quad 14 \end{array}$$

Upon reflection on action I stated this, *"multiplying 2 with 0, to get 0 to put under the numerator 1 was suggested by Jeff."* I thought of asking them to look at the next digit if the first digit of the numerator is smaller than the denominator. Thus instead of finding what number multiplied by 2 will give them 1 or a number smaller than 1, they should have try to find the number that will give them 14 when multiplied by 2. However, since they accepted Jeff's suggestion as logical, I found the need to suggest alternative suggestion unnecessary.

We did a few more questions on both format before I gave them 5 questions to be done individually. I found out that once they mastered the placement of numbers in the long division, they did not take long to acquire this skill.

Day 5

The next lesson was spent giving them mixed problems (problems with remainders and story problem). Most of the pupils were able to do the calculation because they had mastered the long division. Picture 1 illustrated the sample answers provided by Cas.



3. Find the remainder when 628 is divided by 8.

remainder = 4

628 ÷ 8 = 78 (R) 4

4. A taxi can carry 4 passengers. How many taxis are needed to carry 220 passengers?

Number sentence : 220 ÷ 4 = 55

55 taxis are needed to carry 220 passengers.

5. Zamri buys 80 biscuits. If there are 5 biscuits in each packet, how many packets of biscuits does he buy?

Number sentence : 80 ÷ 5 = 16

Picture 1: Sample answers by Cas

Nonetheless, Florence, Fiona and Jack were not able to acquire the symbol representation of division. They did not progress after the first lesson. Lina, George and Ann needed a lot of help. And until the writing of this report, Lina and Ann still need manipulative or drawings to help them do division. They still have not mastered the long division. At one time, George was able to do the long division, but because he frequently missed classes, his progress stalled.

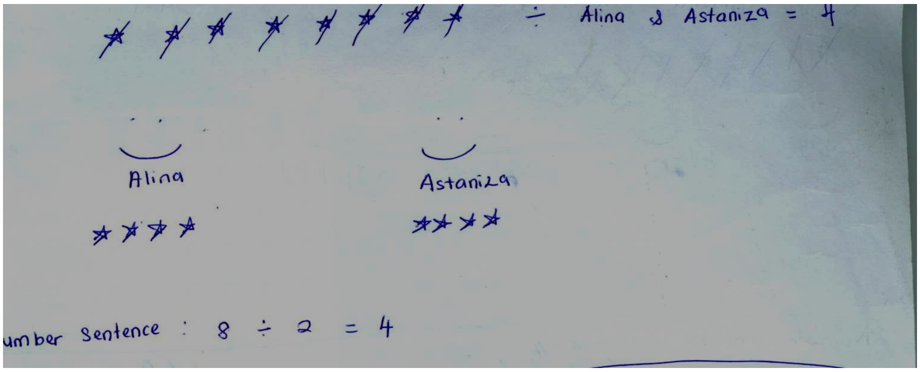
And finally…

I interviewed both Lina and Ann on the 11th of June 2007. Since I could not ask them directly whether they were able to do division, I gave them division questions on the simplest manner, as shown below.

Question 1

I asked verbally: *I have 8 stars. I want to divide the stars equally between Alina and Astaniza. How many will each of them get?*

Then I wrote the following.

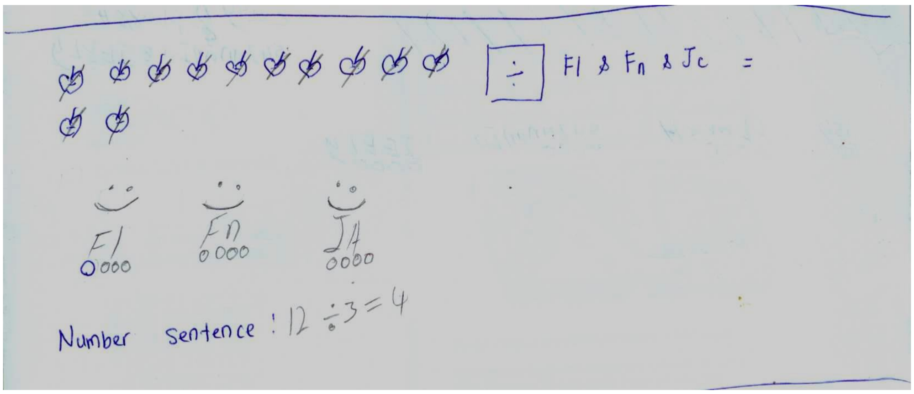


They answered and I wrote their answers down.

Question 2

I asked verbally: *I have 12 apples. I want to divide the apples equally between Florence, Fiona and Jack. How many will each of them get?*

Then I wrote:

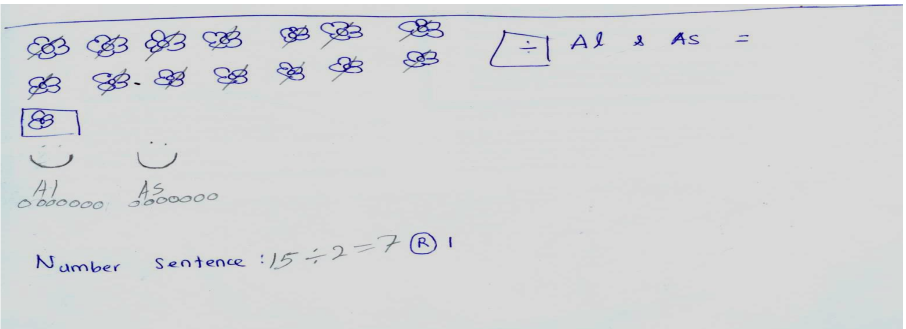


The answers were written by Ann.

Question 3

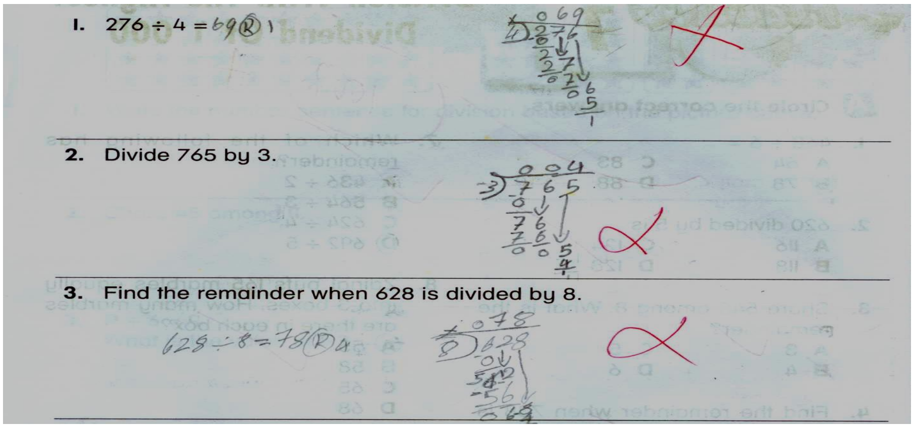
I asked verbally: *I have 15 flowers. I want to divide the flowers equally between Al and As. How many will each of them get?*

Then I wrote:



The answer was written by Lina.

I asked them many questions. But for the purpose of this study, I only include parts of them. Through their responses, I realized that they knew that division is sharing equally. Both Lina and Ann were also able to state and write the term and symbol of remainder correctly. They were also able to transfer questions from word-problem to number sentence and transfer number sentence to the vertical form of division. Picture 2 showed Ann's sample answers.



Picture 2: Sample answers by Ann

However, Ann and Lina were not able to place the numbers correctly when doing long division which was the problem faced by most pupils at the beginning of the lesson.

REFLECTION

A teacher has many concerns. At the beginning of this study, I was merely concern over my pupils' view on mathematics. Yet, after doing the study, I was plague with more questions than answers. I wondered about Florence, Fiona and Jack performances. Ann and Lina's progress baffled me too.

Letting go of Florence, Fiona and Jack to special remedial class is also a bitter sweet experience. On one hand, I felt like abandoning them - giving up on them but on the other hand, I felt relieve to let them go to someone with the expertise.

However, I am happy that at least both Lina and Ann were aware of the meaning of division and division with remainder other than being able to at least use parts of the method I taught (though not all). At least, in teaching them division with remainder by going through lots of real life situations with them first, had enable them to have a firm foundation on the topic. Somehow, I felt better for them to see division with remainder through these lenses rather than being able to do the long division without really knowing what they are for. But of course it would have been better if they were able to comprehend both the meaning and the representation.

Kamii (1990) statements that*, "children who can think, that is, those who can do the logico-math part of arithmetic, can easily learn to write mathematical symbols"*, did not really come to pass in my class too. We still work hard during our initial introduction to long division even though we had went through a lot of story problems. Nevertheless, I much preferred the 'labouring' part knowing that my pupils knew what they were struggling for. Instead of working aimlessly, not really comprehending, the experience would certainly be like stumbling on rocky path with 'blind folded eyes.'

The writing stage of this research also revealed a lot of short comings of this study to me, short comings in both pedagogical and research methodology that I had applied. For instance, given the opportunity, I would like to further verify my data by interviewing **all** of my pupils instead of only selecting a few (I merely assumed that the rest were able based on their work). I would also love to have someone to collaborate with, to have transcripts and coding of the interviews, to use more hands on materials, more group work…the list is endless.

However, I am very happy to be able to reach this stage. I had used more resources than I thought possible, dug deeper than I had ever did. Yet my story is never-ending. As long as I am teaching, I will always have questions… Some I may be able to seek for the answers, but more will remain unanswered. But at this point, I am so glad to be able to put a 'period' to this part of my story. Nevertheless, questions remain. Was I able to lessen the gap between mathematics as a theory and mathematics in real life context in my class? Was I able to help my students view mathematics in a more positive way? Well, that I leave to you, to decide on the answers.

Note: All names have been changed for privacy. This research was carried out at SK Tema in 2007.

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